

THE PENTAGON

A Mathematics Magazine for Students

Volume 79 Number 2

Spring 2020

Contents

<i>Kappa Mu Epsilon National Officers</i>	3
Evenness of Continuous Rhythm Patterns <i>Jake Wellborn, Eric Miles</i>	4
Expanding the “Quadrature of the Parabola” to Cubics using Vieta’s Formulas <i>Benjamin Junkins</i>	20
<i>The Problem Corner</i>	26
<i>Citation for Dr. Rhonda McKee</i> <i>The George R. Mach Distinguished Service Award Recipient</i>	38
<i>Report of the 42nd Biennial Convention</i>	39
<i>Kappa Mu Epsilon News</i>	65
<i>Active Chapters of Kappa Mu Epsilon</i>	84

© 2020 by Kappa Mu Epsilon (<http://www.kappamuepsilon.org>). All rights reserved. General permission is granted to KME members for non-commercial reproduction in limited quantities of individual articles, in whole or in part, provided complete reference is given as to the source. Typeset in Scientific Word and WinEdt. Printed in the United States of America.

The Pentagon (ISSN 0031-4870) is published semiannually in December and May by Kappa Mu Epsilon. No responsibility is assumed for opinions expressed by individual authors. Papers written by undergraduate mathematics students for undergraduate mathematics students are solicited. Papers written by graduate students or faculty will be considered on a space-available basis. Submissions should be made by means of an attachment to an e-mail sent to the editor. Either a TeX file or Word document is acceptable. An additional copy of the article as a pdf file is desirable. Standard notational conventions should be respected. Graphs, tables, or other materials taken from copyrighted works MUST be accompanied by an appropriate release form from the copyright holder permitting their further reproduction. Student authors should include the names and addresses of their faculty advisors. Contributors to The Problem Corner or Kappa Mu Epsilon News are invited to correspond directly with the appropriate Associate Editor.

Editor:

Doug Brown
Department of Mathematics
Catawba College
2300 West Innes Street
Salisbury, NC 28144-2441
dkbrown@catawba.edu

Associate Editors:The Problem Corner:

Pat Costello
Department of Math. and Statistics
Eastern Kentucky University
521 Lancaster Avenue
Richmond, KY 40475-3102
pat.costello@eku.edu

Kappa Mu Epsilon News:

Cynthia Huffman
Department of Mathematics
Pittsburg State University
Pittsburg, KS 66762
cjhuffman@pittstate.edu

The Pentagon is only available in electronic pdf format. Issues may be viewed and downloaded for **free** at the official KME website. Go to <http://www.pentagon.kappamuepsilon.org/> and follow the links.

Kappa Mu Epsilon National Officers

Brian Hollenbeck *President*
Department of Mathematics and Economics
Emporia State University
Emporia, KS 66801
bhollenb@emporia.edu

Don Tosh *President-Elect*
Natural and Applied Sciences Department
Evangel University
Springfield, MO 65802
toshd@evangel.edu

Steven Shattuck *Secretary*
School of Computer Science and Mathematics
University of Central Missouri
Warrensburg, MO 64093
sshattuck@ucmo.edu

David Dempsey *Treasurer*
Department of Mathematical, Computing, & Information Sciences
Jacksonville State University
Jacksonville, AL 36265
ddempsey@jsu.edu

Cynthia Huffman *Historian*
Department of Mathematics
Pittsburg State University
Pittsburg, KS 66762
cjhuffman@pittstate.edu

John W. Snow *Webmaster*
Department of Mathematics
University of Mary Hardin-Baylor
Belton, TX 76513

KME National Website:
<http://www.kappamuepsilon.org/>

Evenness of Continuous Rhythm Patterns

Jake Wellborn, *student*
Eric Miles

CO Delta

Colorado Mesa University
Grand Junction, CO, 81501

Abstract

The connection between mathematics and music is a topic that has interested both mathematicians as well as music theorists. One such connection is the study of rhythm patterns. Rhythm patterns can be represented both numerically and geometrically, and we build on previous work by introducing continuous rhythm patterns and studying a natural measure of evenness on these rhythms. In particular, we consider how evenness changes as we deform a continuous rhythm pattern, and find the maximum evenness values for these rhythms.

1. Introduction

There are many aspects of music that can be thought of in a mathematical sense: sound waves, musical set theory, rhythmic patterns, etc. Studying any of these topics can lead to many insights about both music and mathematics. This paper studies the connection between musical rhythm patterns and mathematics.

We follow [2] in studying musical rhythm patterns, their representations, and the concept of evenness. In particular, we introduce continuous rhythm patterns and study a form of evenness defined using pairwise distances between “hits”. Section 5 contains our main results. In particular, Theorem 4 shows that among continuous rhythm patterns with a given number of hits, the evenly spaced rhythm patterns are maximally even. This allows us to define a relative measure of evenness.

2. Representing Discrete Rhythm Patterns

Consider a rhythm pattern with sixteen evenly spaced beats, where some of those beats are “hits” and the others are “rests.” We assume the

speed at which the rhythm is played is consistent but arbitrary. The fixed number of beats (total of the hits and rests) is called the length of the rhythm. Figure 1 shows four ways of representing the same pattern of length sixteen with five hits.

Binary String : 1000100010010010

X-Dot : X...X...X..X..X.

Interval Length : (4, 4, 3, 3, 2)

Binary Necklace :



Figure 1. Four ways of representing the same rhythm cycle

The first representation, the Binary String [4], uses a 1 to represent a hit and a 0 to represent a rest, while the total number of 1's and 0's make up the length of the rhythm. The X-Dot representation [4] is the same as the binary string with X's instead of 1's and dots instead of 0's; this representation is usually easier to read than the Binary String, while giving the same information. The third representation [4] describes the interval between one hit and the next by counting a hit and the following rests leading to the next hit. The fourth representation is a Binary Necklace [3], where each black dot is a hit, and each white dot is a rest; starting at the top of the circle and moving clockwise, we get the same cycle as the others. The Binary Necklace is very helpful in showing these rhythm patterns as cycles, and we will primarily use this representation throughout the rest of the paper. Notice, when we reach the end of the pattern, it restarts back at the beginning in order to continue the cycle.

Rhythm patterns such as the pattern considered above are called discrete rhythm patterns because of the finite number of locations where a hit may be placed. If a discrete rhythm has a fixed length n , there are only n

locations in the rhythm where a hit may be placed.

2.1 Representing Discrete Rhythm Patterns

If one discrete rhythm pattern can be rotated, or “shifted”, in order to match up with another discrete pattern, we consider these two patterns to be equivalent through rotation. In Figure 2, we can see that by rotating the first pattern counterclockwise 90° , it matches the second pattern, and therefore the two patterns are equivalent by rotation.

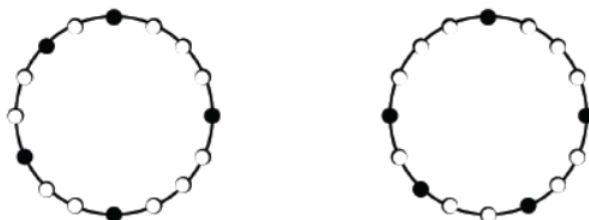


Figure 2. Two DRPs that are equivalent by rotation

3. Continuous Rhythm Patterns

In [4] only discrete patterns are used to represent rhythms, and so far we also have only looked at discrete patterns. However, discrete rhythm patterns (DRPs) have limitations; for example, as we have seen, every discrete pattern has a set number of beats, and therefore only a finite number of locations where a hit may occur. For example, in Figure 3, there are only 16 locations where a hit may occur.



Figure 3. A discrete rhythm pattern

Continuous rhythm patterns (CRPs) can be thought of as a limit of discrete patterns, where we can place hits at any point on a circle. The points of a circle are in a natural bijection with the interval $[0, 1)$. Therefore,

CRPs are also represented on the interval $[0, 1)$, where hits can be placed on any real number in $[0, 1)$. Figure 4 shows two ways of representing the same continuous rhythm pattern. Here we only label hits, while the rests are all values in the spaces between the hits.

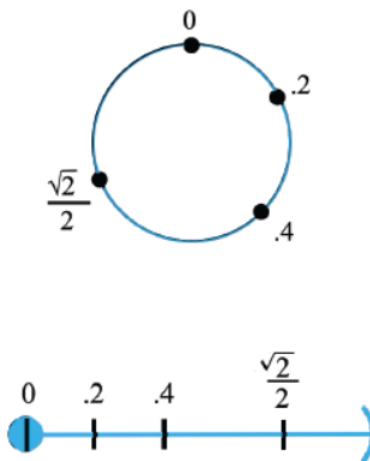


Figure 4. Two ways of representing the same CRP

Note that any discrete rhythm can be represented as a continuous rhythm (see Figure 5), and so we are not losing the ability to look at any discrete rhythms. Also, as with discrete patterns, if one CRP can be rotated in order to match another CRP, then those two patterns are considered equivalent.

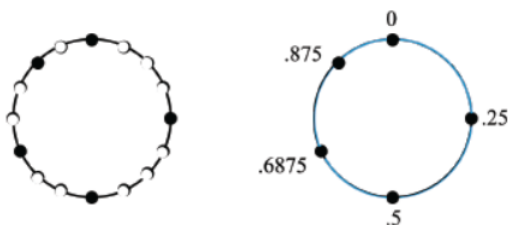


Figure 5. Representing a DRP as a CRP

4. Measuring Distance and Evenness

The concept of evenness, in a musical sense, is about how spread out the hits are in a rhythm pattern [1]. In [4], several methods for quantifying evenness are described. The method that we will use measures the distance between each pair of points (i.e., pairs of hits), and takes the sum of all of

these distances.

We begin by defining the distance between any two hits in a rhythm pattern.

Definition 1 For a CRP $C = \{x_1, x_2, \dots, x_h\} \subseteq [0, 1)$, the distance between hits x_i and x_j is $d(x_i, x_j) = \min\{|x_i - x_j|, 1 - |x_i - x_j|\}$.

We can see in Figure 6 that the shortest distance between the hit at point 0 and the hit at point 0.75 is $d(0, .75) = 0.25$. Note that this definition of distance implies we cannot have a distance greater than 0.5.

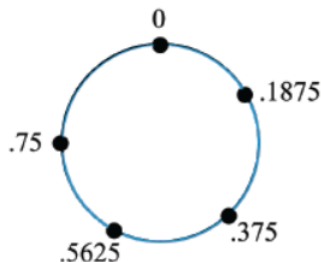


Figure 6. CRP with $d(0, .75) = .25$

Now that we know how to find the distance between any two hits in a CRP, we can start to calculate how evenly spaced the hits in a CRP are, and thus calculate evenness.

Definition 2 For a CRP $C = \{x_1, x_2, \dots, x_h\} \subseteq [0, 1)$, the evenness value for C is

$$\text{evenness}(C) = \sum_{i < j} d(x_i, x_j)$$

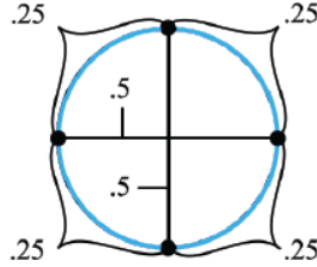
where the sum is over all distinct pairs $x_i, x_j \in C$, with $i < j$. If $h = 1$, we define $\text{evenness}(C) = 0$.

For example, the evenness value of the CRP, C , in Figure 7 is 2.

5. Properties of CRPs

Intuition suggests that when the hits in a CRP are evenly spaced (distance between each set of consecutive hits is equivalent), such a CRP will have the maximum evenness value (the largest possible evenness value for a CRP with the same number of hits). As a first step towards proving this we find the evenness values of evenly spaced CRPs. The formulas for these values will depend on whether we have an odd or even number

of hits. Figure 8 will be a helpful reference for the proof of Theorem 1, where h is odd.



$$\text{evenness}(C) = .25 + .25 + .25 + .25 + .5 + .5 = 2$$

Figure 7. Calculating evenness on a particular CRP

Theorem 1 *If $h = 2k + 1$ for some integer $k \geq 0$, the evenness value for an evenly spaced CRP, C , with h hits is $\text{evenness}(C) = \frac{h^2-1}{8}$.*

Proof. If $h = 1$ (so $k = 0$), we have by definition that $\text{evenness}(C) = 0$. Note that $\frac{1^2-1}{8} = 0$ so the result holds in this case.

Now consider an evenly spaced CRP, C , with h hits where $h = 2k + 1$ for some positive integer k . Since C is evenly spaced, each pair of consecutive hits is separated by a distance of $1/h$. Additionally, from the definition of distances in CRPs, the largest possible distance between two hits is $1/2$. Since $h = 2k + 1$, we have that $\frac{k}{h} < \frac{1}{2}$, and $\frac{k}{h} + \frac{1}{h} > \frac{1}{2}$, and therefore the largest distance between two hits is k/h .

Now, let i be an arbitrary hit in C (we will abbreviate x_i by i). We will begin counting the forward distances from hit i up to hit $i + k$. Note that any distance can be written as a forward distance, since $d(i, j) = d(j, i)$. Furthermore, since our largest distance is k/h , any forward distance can be written as $d(i, i + \epsilon)$ for some integer ϵ with $1 \leq \epsilon \leq k$. Thus, adding these forward distances from each hit i in C ,

$$\sum_{i=1}^h \sum_{\epsilon=1}^k d(i, i + \epsilon)$$

adds all distances between hits. (Here the addition $i + \epsilon$ is done modulo h , so for example, in Figure 8 where $h = 7$ we have $d(6, 6 + 2) = d(6, 1)$.)

Thus $\text{evenness}(C) = \sum_{i=1}^h \sum_{\epsilon=1}^k d(i, i + \epsilon)$. Also, for $1 \leq \epsilon \leq k$, we have $d(i, i + \epsilon) = \frac{(i+\epsilon)-i}{h} = \frac{\epsilon}{h}$.

By rearranging, we have

$$\begin{aligned}
 \text{evenness}(C) &= \sum_{i=1}^h \sum_{\epsilon=1}^k d(i, i + \epsilon) \\
 &= \sum_{\epsilon=1}^k \sum_{i=1}^h d(i, i + \epsilon) \\
 &= \sum_{\epsilon=1}^k \sum_{i=1}^h \frac{\epsilon}{h} = \sum_{\epsilon=1}^k h \left(\frac{\epsilon}{h} \right) \\
 &= \sum_{\epsilon=1}^k \epsilon \\
 &= \frac{k(k+1)}{2} = \frac{h^2 - 1}{8}.
 \end{aligned}$$

■

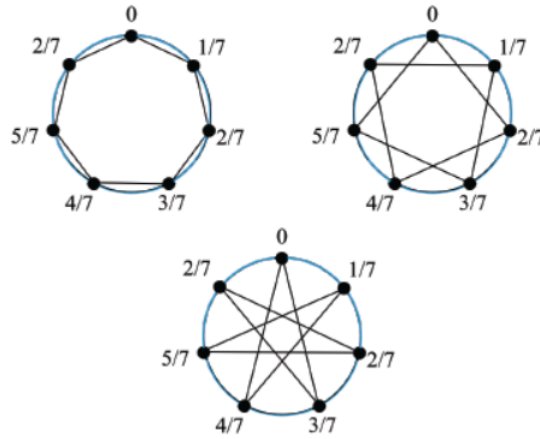


Figure 8. Evenness value for an evenly spaced CRP C with 7 hits is $7\left(\frac{1}{7}\right) + 7\left(\frac{2}{7}\right) + 7\left(\frac{3}{7}\right) = 1 + 2 + 3 = 6$.

The proof for when we have an even number of hits is similar, but we have to be more careful not to double-count any distances (see Figure 9).

Theorem 2 *If $h = 2k$ for some integer $k \geq 1$, the evenness value for an evenly spaced CRP, C , with h hits is $\text{evenness}(C) = h^2/8$.*

Proof. Consider an evenly spaced CRP, C , with h hits, where $h = 2k$, for

some positive integer k . Since C is evenly spaced, each pair of consecutive hits is separated by a distance of $1/h$. Additionally, from the definition of distances in CRPs, the largest possible distance between two hits is $1/2$. Since $h = 2k$, we have that $\frac{k}{h} = \frac{1}{2}$, and therefore the largest distance between two hits is k/h .

As in Theorem 1, any distance can be written as $d(i, i + \epsilon)$ for some integer ϵ where $1 \leq \epsilon \leq k$ and adding these forward distances from each hit i in C ,

$$\sum_{i=1}^h \sum_{\epsilon=1}^k d(i, i + \epsilon)$$

adds all distances between hits. (As in Theorem 1, the addition $i + \epsilon$ is done modulo h .)

In our sum above, some distances are counted twice (the cases where $\epsilon = k$). Thus, our summation with duplicates removed is

$$\text{evenness}(C) = \sum_{i=1}^h \sum_{\epsilon=1}^{k-1} d(i, i + \epsilon) + \sum_{i=1}^k d(i, i + k).$$

Note that for $1 \leq \epsilon \leq k$ we have $d(i, i + \epsilon) = \frac{(i+\epsilon)-i}{h} = \frac{\epsilon}{h}$. By rearranging our sum, we have

$$\begin{aligned} \text{evenness}(C) &= \sum_{\epsilon=1}^{k-1} \sum_{i=1}^h d(i, i + \epsilon) + \sum_{i=1}^k d(i, i + k) \\ &= \sum_{\epsilon=1}^{k-1} \sum_{i=1}^h \frac{\epsilon}{h} + \sum_{i=1}^k \frac{1}{2} \\ &= \left(\sum_{\epsilon=1}^{k-1} h \left(\frac{\epsilon}{h} \right) \right) + \frac{k}{2} \\ &= \left(\sum_{\epsilon=1}^{k-1} \epsilon \right) + \frac{k}{2} \\ &= \frac{(k-1)k}{2} + \frac{k}{2} \\ &= \frac{k^2}{2} \\ &= \frac{h^2}{8}. \end{aligned}$$

■

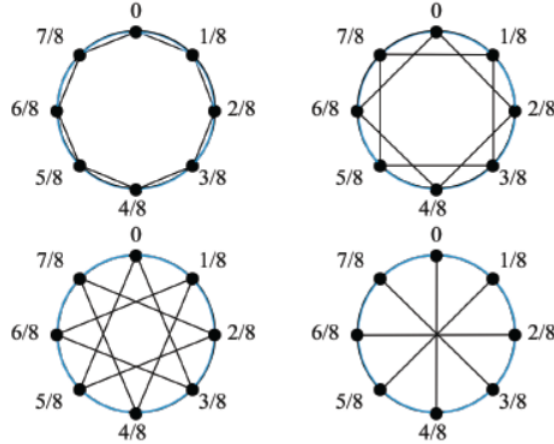


Figure 9. For an evenly spaced CRP C with $h = 8$ hits, we can see that there are 8 distances of $\frac{1}{8}$, 8 distances of $\frac{2}{8}$, 8 distances of $\frac{3}{8}$, and 4 distances of $\frac{4}{8}$, which gives $\text{evenness}(C) = 8(\frac{1}{8}) + 8(\frac{2}{8}) + 8(\frac{3}{8}) + 4(\frac{4}{8}) = 1 + 2 + 3 + 2 = 8$.

Table 1 lists several evenness values for evenly spaced CRPs given h , the number of hits.

h	evenness	h	evenness
1	0	6	4.5
2	0.5	7	6
3	1	8	8
4	2	9	10
5	3	10	12.5

Table 1. Evenness values for evenly spaced CRPs with number of hits $h = 1, \dots, 10$.

In order to show that evenly spaced CRPs are maximally even (that is, have the maximum possible evenness value), we study how evenness changes as we alter a CRP.

Lemma 1 For a CRP, C , with $h \geq 3$ hits, if there exists two hits, c_x and c_y , such that $d(c_x, c_y) = 1/2$, then $\text{evenness}(C) = \text{evenness}(C') + \frac{h-1}{2}$, where $C' = C \setminus \{c_x, c_y\}$.

Proof. Consider a CRP C_h with $h \geq 3$ hits. Suppose there exist two hits in C_h , say c_x and c_y , such that $d(c_x, c_y) = 1/2$. Now, consider another hit,

c_z , such that $c_x \neq c_z \neq c_y$. Since $d(c_x, c_y) = 1/2$, we have $d(c_x, c_z) + d(c_z, c_y) = 1/2$. Because c_z is arbitrary, this is true for every hit in C_h other than c_x and c_y . Therefore, we have that the sum of all distances in C_h involving one or both hits c_x, c_y is

$$d(c_x, c_y) + \sum_{\substack{c_z \\ c_z \neq c_x, c_y}} (d(c_x, c_z) + d(c_z, c_y)) = \frac{1}{2} + (h-2)\frac{1}{2} = \frac{h-1}{2}.$$

Lastly, since the evenness value for C_h is the sum of all distances in C_h , we can say that the evenness value of C_h is the sum of all distances in C_h that involve c_x and/or c_y , added to the sum of all distances in C_h that do not involve either c_x or c_y . Hence, $\text{evenness}(C_h) = \text{evenness}(C_{h-2}) + \frac{h-1}{2}$, where $C_{h-2} = C_h \setminus \{c_x, c_y\}$. ■

Remark 3 *In the results that follow, we use the language of “moving” a hit c of a CRP, C . By this, we simply mean that the original CRP C which contains c is replaced by a new CRP C' in which the hit c is replaced by a new hit c' .*

Lemma 2 *For a CRP C with h hits ($h \geq 2$), there exists a hit that is already at a distance $1/2$ from another hit, or can be moved to where it is at a distance $1/2$ from another hit, and through this movement the evenness value of C will either increase or stay the same.*

Proof. Consider a CRP $C_h = \{c_1, c_2, \dots, c_h\} \subseteq [0, 1)$ where $c_i \leq c_{i+1}$. Without loss of generality, we may shift C_h so that $c_1 = 0$. If there exists a hit at the point $1/2$, we are done: we will not change our CRP and therefore the evenness does not change. So suppose there does not exist a hit in C_h at the point $1/2$. If there are only $h = 2$ hits, we may simply move the hit c_2 to $c'_2 = 1/2$ and again we are done since

$$\text{evenness}(C) = d(c_1, c_2) < 1/2 = d(0, 1/2) = \text{evenness}(C')$$

where we use C' to represent our adjusted CRP.

Thus we may assume $h \geq 3$ (recall we are still assuming that there does not exist a hit in C_h at the point $1/2$) and separate the hits in C besides $c_1 = 0$ into two sections: those hits in the interval $(0, 1/2)$ and those in $(1/2, 1)$. Between these two sets choose the set that contains more hits. If the sets contain the same number of hits then it does not matter which set is chosen. Now, call the chosen set of hits A , and call the other set of hits B (note that B could be the empty set), and call c_k the hit in set A that is closest to the point $1/2$ (see Figure 10). Let us assume that set A is the set of hits in the interval $(0, 1/2)$ (the other case is symmetric). Move hit c_k to the

point $1/2$ and rename the point $c'_k = 1/2$, so $d(c_1, c'_k) = d(0, 1/2) = 1/2$. Call the resulting CRP \tilde{C} . Now, suppose $d(c_k, c'_k) = x$.

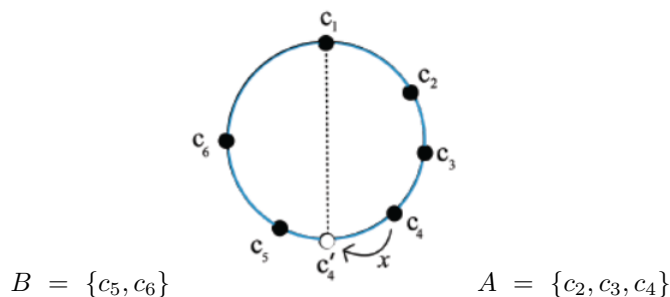


Figure 10. Since there are more hits to the right of $c_1 + \frac{1}{2}$, we move c_4 by a distance of x .

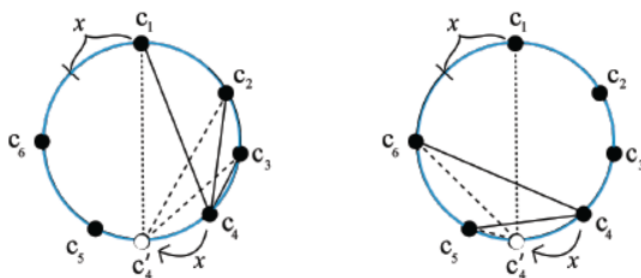


Figure 11. When there are no hits in $(1 - x, 1)$ and we move c_4 to c'_4 , distances from hits in A increase by x , but distances from hits in B decrease by x .

We will now count the distances from hit $c'_k = 1/2$ to all other hits in C_h . Since $d(c_1, c'_k) = 1/2 > d(c_1, c_k)$, the distance from hit $c_1 = 0$ to c_k increased by the amount x through our shift. Now, let c_i be any hit in A other than c_k . Then $1/2 = d(c_1, c'_k) = d(c_1, c_k) + x$, and $d(c_i, c'_k) = d(c_i, c_k) + x$. Therefore, the distance from every hit in $(A \setminus \{c_k\}) \cup \{c_1\} = \{c_1, c_2, \dots, c_{k-1}\}$ to c_k increased by the amount x through our shift (see Figure 11).

If B is the empty set we are done since $\text{evenness}(C) < \text{evenness}(\tilde{C})$, as each distance between hits in C has, through our shift, either remained the same (when c_k is not involved) or has increased by x . Now suppose that B is nonempty and for $B = \{c_{k+1}, c_{k+2}, \dots, c_h\}$, there are $b = h - k$ hits in B , and from the way we chose sets A and B we have $b \leq k$. Let c_j be

any hit in B . First, suppose there are no hits in the interval $(1-x, 1)$. Then $d(c'_k, c_j) = d(1/2, c_j) = d(c_k, c_j) - x$. Therefore, the distance from every hit in B to hit c_k decreased by the length x through our shift. Now, from the way we chose our sets, we know $(A \setminus \{c_k\}) \cup \{c_1\} = \{c_1, c_2, \dots, c_{k-1}\}$ has either more hits or the same number of hits as the set B , and since our shift caused the distance from c_k to every hit in $\{c_1, c_2, \dots, c_{k-1}\}$ to increase by x , we have that $\text{evenness}(C) \leq \text{evenness}(\tilde{C})$, and this case is done.

Now, suppose there are $m \geq 1$ hits from B that are in the interval $(1-x, 1)$, and note $m \leq b$. Let c^* be a hit in the interval $(1-x, 1)$, and let $d(c^*, c_1) = d(c^*, 0) = y$. Since $c^* \in (1-x, 1)$, we know $y < x$ (see Figure 12). In order to find the change in distance from c_k to c^* through our shift, we will calculate $d(c_k, c^*) - d(c'_k, c^*)$.

First note that

$$d(c_k, c^*) = d(c_1, c_k) + d(c_1, c^*) = \left(\frac{1}{2} - x\right) + y.$$

Next, since $c_1 = 0$ and $c'_k = 1/2$ we have

$$d(c'_k, c^*) = d(c_1, c'_k) - d(c_1, c^*) = \frac{1}{2} - y.$$

Combining these yields

$$\begin{aligned} d(c_k, c^*) - d(c'_k, c^*) &= \left(\frac{1}{2} - x + y\right) - \left(\frac{1}{2} - y\right) \\ &= -x + 2y \\ &< x \end{aligned}$$

where the last inequality follows since $y < x$. (Note that the quantity $-x + 2y$ above could be negative.)

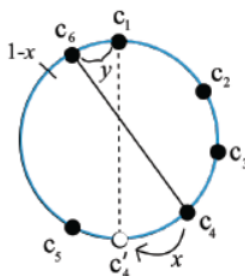


Figure 12. When there are hits in $(1-x, 1)$ and we move c_k to c'_k , distances to the hits in $(1-x, 1)$ can increase or decrease, but will only decrease by an amount less than x .

Since $d(c_k, c^*) - d(c'_k, c^*) < x$, the distance from c_k to c^* through our shift either increased or decreased by an amount less than x .

Therefore, for the m hits in the interval $(1 - x, 1)$, the distance from each of those hits to c_k through our shift either increased or decreased by an amount less than x .

Summarizing, our shift caused each distance from the m (where $m \leq b$) hits in $(1 - x, 1)$ to c_k either to increase, or to decrease by an amount less than x , and caused the distance from any hits in $B \setminus (1 - x, 1)$ to c_k to increase by x . Thus we may say that our shift caused the distance from the b hits in B to c_k to increase, or to decrease by an amount less than x (possibly a negative amount).

Then, since our shift caused the distance from every hit in $(A \setminus \{c_k\}) \cup \{c_1\} = \{c_1, c_2, \dots, c_{k-1}\}$ to c_k to increase by x , and since there are either more hits or the same number of hits in $(A \setminus \{c_k\}) \cup \{c_1\}$ than there are in B , it follows that $\text{evenness}(C) < \text{evenness}(\tilde{C})$ as desired. ■

Now we are ready to prove that evenly spaced are maximally even.

Theorem 4 *Let C be a CRP with $h \geq 1$ hits, and let S be an evenly spaced CRP with h hits. Then, $\text{evenness}(C) \leq \text{evenness}(S)$.*

Proof. We proceed by induction.

Base Cases ($h = 1$ and 2): Consider an arbitrary CRP, C , with 1 hit. Since there is only 1 hit, it is already evenly spaced. Therefore, the evenness value of C is equal to that of an evenly spaced CRP with 1 hit. Now, suppose C has 2 hits, c_1 and c_2 . Since there are only two hits, $\text{evenness}(C) = d(c_1, c_2)$. Since the largest possible distance is $\frac{1}{2}$, the largest evenness value for a CRP with two hits is $\frac{1}{2}$, which is an evenly spaced CRP. Therefore, the evenness value for C is less than or equal to the evenness value of an evenly spaced CRP with 2 hits.

Inductive Step: Let $h \geq 3$. Suppose that for any CRP C_{h-2} with $h - 2$ hits, we have $\text{evenness}(C_{h-2}) \leq \text{evenness}(S_{h-2})$, where S_{h-2} is the evenly spaced CRP with $h - 2$ hits.

From Lemma 2, we know that there exists a hit $c_k \in C = \{c_1, c_2, \dots, c_h\}$ such that by shifting hit c_k to the position $c'_k = c_1 + \frac{1}{2}$ and calling the resulting CRP \tilde{C}_h , we have $\text{evenness}(C_h) \leq \text{evenness}(\tilde{C}_h)$. Now, since we have two points that are distance $\frac{1}{2}$ from each other, namely c_1 and c'_k , Lemma 1 tells us that $\text{evenness}(\tilde{C}_h) = \text{evenness}(\tilde{C}_{h-2}) + \frac{h-1}{2}$, where $\tilde{C}_{h-2} = \tilde{C}_h \setminus \{c_1, c_k\}$.

Now, from our inductive hypothesis, since \tilde{C}_{h-2} has $h - 2$ hits, we know $\text{evenness}(\tilde{C}_{h-2}) \leq \text{evenness}(S_{h-2})$. Therefore we have that

$$\begin{aligned}
\text{evenness}(C_h) &\leq \text{evenness}(\tilde{C}_h) \\
&= \text{evenness}(\tilde{C}_{h-2}) + \frac{h-1}{2} \\
&\leq \text{evenness}(S_{h-2}) + \frac{h-1}{2}
\end{aligned}$$

To complete the inductive step it is enough to prove that

$$\text{evenness}(S_{h-2}) + \frac{h-1}{2} = \text{evenness}(S_h).$$

We have two cases: S_{h-2} has an odd number of hits, or S_{h-2} has an even number of hits.

Case 1: Suppose S_{h-2} has an odd number of hits, so $h = 2k + 1$ for some integer $k \geq 1$. Then $h - 2 = 2k - 1 \geq 1$ and from Theorem 1 we know $\text{evenness}(S_{h-2}) = \frac{(h-2)^2-1}{8}$. Thus

$$\begin{aligned}
\text{evenness}(S_{h-2}) + \frac{h-1}{2} &= \frac{(h-2)^2-1}{8} + \frac{h-1}{2} \\
&= \frac{h^2-4h+3}{8} + \frac{4h-4}{8} \\
&= \frac{h^2-1}{8} \\
&= \text{evenness}(S_h).
\end{aligned}$$

Case 2: Suppose S_{h-2} has an even number of hits, so $h = 2k$ for some integer $k \geq 2$. Then $h - 2 = 2k - 2 \geq 2$ and from Theorem 2 we know $\text{evenness}(S_{h-2}) = \frac{(h-2)^2}{8}$. Thus

$$\begin{aligned}
\text{evenness}(S_{h-2}) + \frac{h-1}{2} &= \frac{(h-2)^2}{8} + \frac{h-1}{2} \\
&= \frac{h^2-4h+4}{8} + \frac{4h-4}{8} \\
&= \frac{h^2}{8} \\
&= \text{evenness}(S_h).
\end{aligned}$$

Hence $\text{evenness}(C_h) \leq \text{evenness}(S_h)$, as desired. ■

By combining Theorems 1, 2, and 4, we obtain the following corollary. Here we define $\text{MaxEvenness}(h)$ for $h \in \mathbb{N}$ as the maximum value that $\text{evenness}(C)$ can take for a CRP, C , with h hits.

Corollary 1 *For odd h , $\text{MaxEvenness}(h) = \frac{h^2-1}{8}$, and for even h we have $\text{MaxEvenness}(h) = \frac{h^2}{8}$.*

6. Further Research

We conclude by defining a notion of relative evenness and briefly discuss possible directions for research.

Because we know the largest possible evenness value for a CRP with any number of hits, $h \geq 1$, we can define a relative measure of evenness.

Definition 3 *For a CRP C , with $h \geq 1$ hits, the relative evenness value of C is*

$$\text{RelEvenness}(C) = \frac{\text{evenness}(C)}{\text{MaxEvenness}(h)}.$$

Relative evenness values give a more meaningful measure of how evenly spaced a rhythm is than an absolute measure such as that in Definition 2, as relative values lie on a strict scale from 0 to 1. Hence, we know that the closer any CRPs relative evenness value is to 1, the closer it is to being maximally even. One could use this relative notion of evenness to conduct research following [4]. More specifically, trends of music in different cultures could be analyzed to see if different cultures prefer rhythms that are in a specific range of relative evenness.

Other avenues of research are more abstract. For example, Definition 2, evenly spaced CRPs are not the only CRPs that are maximally even. In fact there are infinitely many CRPs that have maximum evenness that are not evenly spaced. (As an example, if a CRP has two hits that are a distance of $1/2$ apart, the evenness of a CRP is not changed if we move those hits while keeping their distance at $1/2$. Thus, in any evenly spaced CRP with an even number of hits, we may move any pair of hits a distance of $1/2$ apart and maintain the same evenness.) One could study what types of CRPs are maximally even, and what other properties they have in common.

Finally, we described in Section 2.1 the notion of rhythm patterns being equivalent under the operation of rotations, or “shifts.” It would be interesting to study the equivalence classes and parameter spaces of these classes for CRPs with particular numbers of hits.

References

- [1] Steven Block and Jack Douthett, “Vector Products and Intervallic Weighting,” *Journal of Music Theory* Vol. 38, 1994, pp. 21 – 41.
- [2] Eric Demaine, Francisco Gomez-Martin, Henk Meijer, David Rapaport, Perouz Taslakian, Godfried Toussaint, Terry Winograd and David Wood, “The Distance Geometry of Music,” *Computational Geometry: Theory and Applications*, 2009.
- [3] Rachel W. Hall and Paul Klingsberg, “Asymmetric Rhythms, Tiling Canons, and Burnside’s Lemma,” in *Bridges: Mathematical Connections in Art, Music, and Science*, (R. Sarhangi and C. Sequin, editors), Winfield, Kansas, 2004, pp. 189 – 194.
- [4] Godfried Toussaint, “The Geometry of Musical Rhythm,” *Discrete and Computational Geometry*, 2004, pp. 198 – 212.

Expanding the “Quadrature of the Parabola” to Cubics using Vieta’s Formulas

Benjamin Junkins *student*

AL Theta

Jacksonville State University
Jacksonville, AL, 36265

Acknowledgement: Thank you, Dr. Jadeok Kim for teaching and mentoring me throughout my study as an undergraduate student. Without you constantly challenging me, this would not have been possible. I will never forget what it means to be a mathematician because of you: to have fun trying to solve what seems impossible.

Abstract

A brief summary of Archimedes’ “Quadrature of the Parabola” is given showing the use of recursively defined triangles to find the area bounded by a parabola and a line. A special relationship between the initial inscribed triangle and the total bounded area was shown in his work, and this idea of finding a relationship between the initial triangle and bounded area is then expanded to a general depressed cubic function. The derivation of Vieta’s formulas is shown, and the formulas are used to greatly simplify the algebra and calculus of this expansion. After finding the relationship, a pattern from the quadratic to cubic case is found, and it is conjectured that this pattern holds for any n th degree polynomials of a certain depressed form.

1. Introduction

How to find the exact area bounded by two curves was an important question throughout the history of mathematics for centuries before Newton and Leibniz. For the case of two lines, it is trivial to find the area between the curves since it will either be a triangle or rectangle, but for higher order polynomials and other functions, it was near impossible. In the 3rd century B.C.E., 1500 years before Newton and Leibniz, Archimedes al-

ready solved this question for the case of parabola and a line using the sum of an infinite number of recursively inscribed triangles in a letter written to his friend Dositheus, discussed in Section 4. He found that the exact area bounded by a line and parabola is $\frac{4}{3}X$, where X is the area of the inscribed triangle with a base that is the line segment intersecting the parabola, and a third vertex that is the same as the vertex for the parabola. The question asked here is whether Archimedes' idea, of having the bounded area expressed as a ratio of the inscribed triangle, can be expanded to cubic functions and higher order polynomials. Cubic equations of the form $y = ax^3 - bx$ with $a, b > 0$, referred to as depressed cubics, are used so a triangle can be inscribed between a tangent line and the cubic. The exact area bounded by a cubic and a tangent line is found and then compared to the area bounded by the inscribed triangle. Vieta's formulas provide convenient equations since the coefficient for x^2 is zero, so they are used to make the integration and algebra easier when determining the bounded area of a depressed cubic.

2. Quadrature of the Parabola

Archimedes wrote in his *Quadrature of the Parabola* that the area bounded

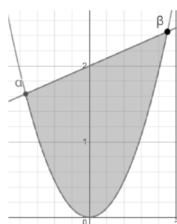


Figure 1.

by a line intersecting a parabola at α and β (Figure 1) is $\frac{4}{3}X$, where X is the area of the triangle inscribed in the curve with vertices α , β , and the vertex of the parabola (Figure 2). The argument Archimedes used was similar to the Riemann Sum in finding the area of rectangles under a curve and adding the areas of rectangles together. Instead of rectangles, Archimedes used triangles. The initial triangle would be the triangle shown in Figure 2

with area X , and the next triangles would still be inscribed in the parabola with a new vertices at $\frac{\alpha}{2}$ and $\frac{\beta}{2}$ (two new triangles are made) and the remaining vertices are α , β , and the vertex of the parabola (Figure 3).

This process of halving the distance of the previous vertices to make a new one is repeated and infinite number of times. Notice that the number of new triangles created is 2^n , where n is the number of times the distance is halved. Archimedes found that the total area formed by the new triangles is always $\frac{1}{4}$ the previous area. Since there are infinitely many triangles, the argument was made that the area of the parabola is exactly equal to the sum of all the triangles. As an infinite sum, Archimedes found a geometric series of the form

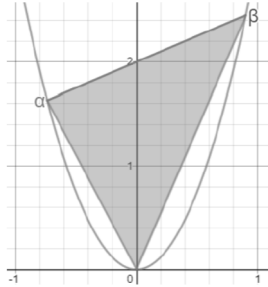


Figure 2.

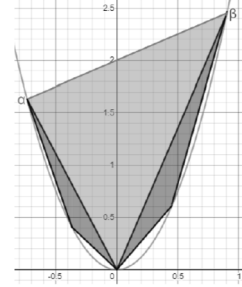


Figure 3.

$$\begin{aligned}
 & X + \frac{1}{4}X + \frac{1}{16}X + \frac{1}{64}X + \dots \\
 &= X \left(1 + \left(\frac{1}{4}\right)^1 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \dots \right)
 \end{aligned}$$

It was known that the geometric series above converges to $\frac{4}{3}X$. [1]

If this method is extended to a cubic of the form $y = ax^3 + bx$ (Figure 4) then there is excess area that must be subtracted. These computations become tedious using the Archimedean method. Instead, Calculus is used with Vieta's Formulas.

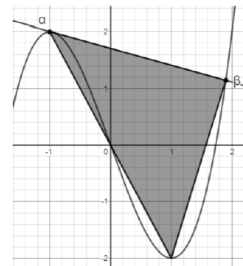


Figure 4.

3. Vieta's Formulas

Vieta's Formulas state that given a polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$$

and if r_1, r_2, \dots, r_n are the roots of P and if $a_n = 1$, then

$$a_{n-1} = -(r_1 + r_2 + \dots + r_n) \quad (1)$$

$$a_{n-2} = r_1r_2 + r_1r_3 + \dots + r_2r_3 + r_2r_4 + \dots + r_{n-1}r_n \quad (2)$$

$$a_{n-3} = -(r_1r_2r_3 + r_1r_2r_4 + \dots + r_{n-2}r_{n-1}r_n) \quad (3)$$

$$\vdots$$

$$a_0 = (-1)^n (r_1r_2\dots r_n) \quad (4)$$

The proof of this applies the Fundamental Theorem of Algebra that says P factors into $(x - r_1)(x - r_2)\dots(x - r_n)$. Expanding this gives

$$x^n - (r_1 + \dots + r_n)x^{n-1} + (r_1r_2 + \dots + r_{n-1}r_n)x^{n-2} - \dots + (-1)^n r_1\dots r_n.$$

Since these two polynomials must be the same, their coefficients must be the same. This yields the above equations. Equations (1), (2), and (4) will be the main results used throughout the discussion.

4. Depressed Cubics

Define a depressed cubic function as $v(x) = px^3 - qx$ where $p, q \in \mathbb{R}^+$. It follows directly from Vieta's Formulas that any function $w(x) = mx + b$, where $m, b \in \mathbb{R}$, that intersects any function $v(x)$ at minimum two distinct points and at most three distinct points will have points of intersection $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$, and $P_3 = (x_3, y_3)$ such that $x_1 + x_2 = -x_3$.

Theorem 1 *If a function $w(x)$ intersects a function $v(x)$ at minimum two distinct points, then $x_1 + x_2 = -x_3$, where x_1, x_2 , and x_3 are where w intersects v .*

Proof. Let $w(x) = mx + b$ intersect a depressed cubic curve $v(x) = px^3 - qx$ at x_1, x_2 , and x_3 . We show that $x_1 + x_2 = -x_3$ using Vieta's Formulas. To find the points of intersection, we set

$$\begin{aligned} px^3 - qx &= mx + b \\ \Rightarrow px^3 - (q - m)x + b &= 0 \\ \Rightarrow x^3 - \frac{(q - m)}{p}x + \frac{b}{p} &= 0. \end{aligned}$$

so it is equivalent to say that w intersects v at least twice if and only if

$x^3 - \frac{(q-m)}{p}x + \frac{b}{p}$ has at least two roots. It follows from (1), since $a_{n-1} = 0$ then $x_1 + x_2 + x_3 = 0$. Thus, we see $x_1 + x_2 = -x_3$. ■

Note that this result allows for a double root. If the intersecting line is tangent to the depressed cubic we have $x_1 + x_1 = 2x_1 = -x_2$.

5. Extension to Depressed Cubics

Consider and a tangent line $y = mx + n$ that intersects this cubic at α and β (Figure 5). Let $\alpha = -p$. Then Theorem 1 gives $\beta = 2p$. The slope of the tangent line is $m = 3ax^2 + b$. The bounded area can now be calculated from the integral

$$\begin{aligned} A_B &= \int_{\alpha}^{\beta} (mx + n) - (ax^3 - bx) dx \\ &= -a \int_{-p}^{2p} \left(x^3 - \frac{(m+b)}{a}x - \frac{n}{a} \right) dx. \end{aligned}$$

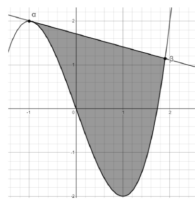


Figure 5.

Note that the roots of the integrand, $-p$ and $2p$, are $-\frac{m+b}{a} = -3p^2$ and $-\frac{n}{a} = -2p^3$ by (2) and (4) respectively, making the integral simplify to

$$A_B = -a \int_{-p}^{2p} (x^3 - 3p^2x - 2p^3) dx.$$

The absolute value of a is used to yield a positive area of

$$\begin{aligned} A_B &= |a| \frac{27}{4} p^4 \\ &= \frac{|a|(2p - (-p))^4}{12} \\ &= \frac{|a|(\beta - \alpha)^4}{12}. \end{aligned} \quad (5)$$

All that is left to find the ratio of the inscribed triangle area to the bounded area of the tangent line, is to find the area of the inscribed triangle and divide this by (5).

Consider the triangle inscribed in the cubic and tangent shown in Figure 3 with vertices $\alpha = -p$ and $\beta = 2p$ defined as above and third vertex defined as $\gamma = p$ to maintain symmetry about to origin for the line $\alpha\gamma$.

This area can be found using the formula

$$A_T = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ y_1 & y_2 & y_3 \\ x_1 & x_2 & x_3 \end{vmatrix}, \quad (6)$$

where the points (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) are the vertices of the triangle. Equation (6) simplifies to

$$A_T = \frac{2a(\beta - \alpha)^4}{27}. \quad (7)$$

The ratio is now $R = \frac{A_B}{A_T} = \frac{9}{8}$.

6. Conclusion

Equation (5) is worth noting since it is a closed formula for the area between a tangent line and a cubic solely based on where the intersection occurs. A similar formula can be derived for a quadratic by using (5) and multiplying by $\frac{4}{3}$ which is

$$\frac{|a|(\beta - \alpha)^3}{6}$$

It is also worth noting that the ratio of the triangle area to the bounded area for a quadratic is

$$\frac{4}{3} = \frac{2^2}{2^2 - 1}$$

and for a cubic is

$$\frac{9}{8} = \frac{3^2}{3^2 - 1}.$$

It can be conjectured that for an n^{th} degree polynomial, the ratio of the inscribed area to the bounded area is

$$\frac{n^2}{n^2 - 1}.$$

Centuries before Newton, Archimedes had the intuition of calculus using triangles instead of rectangles. While the Riemann sum is easier to use in general, a integral defined in terms of recursive triangles could be more useful and would require further research.

References

- [1] Archimedes. (1912). *Quadrature of the Parabola* In T. L. Heath (Ed.), *The Works of Archimedes* (pp. 233–252). New York, NY: Dover Publication, Inc.

The Problem Corner

Edited by Pat Costello

The Problem Corner invites questions of interest to undergraduate students. As a rule, the solution should not demand any tools beyond calculus and linear algebra. Although new problems are preferred, old ones of particular interest or charm are welcome, provided the source is given. Solutions should accompany problems submitted for publication. Solutions of the following new problems should be submitted on separate sheets before April 15, 2021. Solutions received after this will be considered up to the time when copy is prepared for publication. The solutions received will be published in the Spring 2021 issue of *The Pentagon*. Preference will be given to correct student solutions. Affirmation of student status and school should be included with solutions. New problems and solutions to problems in this issue should be sent to Pat Costello, Department of Mathematics and Statistics, Eastern Kentucky University, 521 Lancaster Avenue, Richmond, KY 40475-3102 (e-mail: pat.costello@eku.edu, fax: (859) 622-3051)

NEW PROBLEMS 859 - 869

Problem 859. *Proposed by the editor.*

A regular n -gon is inscribed in a circle which is inscribed in a square with side length x . Find the length of one side of the n -gon in terms of x . [The particular case where $n = 6$ and $x = 25$ was a theMathContest.com problem.]

Problem 860. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Find out whether or not there is a prime p such that $p!$ ends in exactly 2020 zeroes. Is there a corresponding prime q such that $q!$ ends in exactly 2019 zeroes?

Problem 861. *Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.*

Let a_0, a_1, a_2, a_3 be any distinct nonzero real numbers. For any integer $n \geq 3$, prove that the zeroes of the polynomial

$$A(x) = x^{2n} + a_3x^3 + a_2x^2 + a_1x + a_0$$

cannot all be real.

Problem 862. Proposed by Daniel Sitaru, “Theodor Costescu” National Economic College, Drobeta Turnu – Severin, Romania.

If $a, b \in \mathbb{C}$ are such that

$$|a^2 + 25| \leq 5, |b^2 + 36| \leq 6, |a + 5| \leq \sqrt{5}, \text{ and } |b + 6| \leq \sqrt{6}$$

then

$$|a + b|^2 + |a - b|^2 \leq 4.$$

Problem 863. Proposed by Daniel Sitaru, “Theodor Costescu” National Economic College, Drobeta Turnu – Severin, Romania.

If $a, b, c > 0$ and $a + b + c = 6$, then

$$\frac{a^2}{\sqrt{b^2 + 6bc + 5c^2}} + \frac{b^2}{\sqrt{c^2 + 6ca + 5a^2}} + \frac{c^2}{\sqrt{a^2 + 6ab + 5b^2}} \geq \sqrt{3}$$

Problem 864. Proposed by Toyesh Prakash Sharma (student) St. C.F Andrews School, Agra, India.

Evaluate the following sum

$$\sum_{z=0}^{\infty} \sum_{y=0}^{\infty} \sum_{x=0}^{\infty} \frac{x + y + z}{2^{(x+y+z)}}$$

Problem 865. Proposed by Pedro H.O. Pantoja, Natal/RN, Brazil.

Let a, b, c be positive real numbers such that $a + b + c = 1$. Prove that

$$27(a^5 + b^5 + c^5) + 13 \leq 40(a^2 + b^2 + c^2).$$

Problem 866. Proposed by Dorin Marghidanu, Colegiul National ‘A. I. Cuze’, Corabia, Romania.

If a, b, c are the lengths of the sides of a triangle, prove the following

$$\sqrt{\frac{-a + b + c}{a}} + \sqrt{\frac{a - b + c}{b}} + \sqrt{\frac{a + b - c}{c}} > 2\sqrt{2}$$

Problem 867. Proposed by George Stoica, Saint John, New Brunswick, Canada.

Let a, b, c be positive real numbers and $x, y, z \geq 0$ such that

$$x + aby \leq a(y + z), y + bcz \leq b(z + x), \text{ and } z + cax \leq c(x + y).$$

Prove that $x = y = z = 0$. Is the conclusion still true if one assumes that $a, b, c \geq 0$?

Problem 868. Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.

Let the sequence (a_n) be defined by $a_n = \sum_{k=1}^n \arctan\left(\frac{1}{k^2 - k + 1}\right)$. Com-

pute the following:

- (i) $\lim_{n \rightarrow \infty} a_n = a$;
- (ii) $\lim_{n \rightarrow \infty} (a_n - a)n$.

Problem 869. Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.

If $\gamma_n = -\ln n + \sum_{k=1}^n \frac{1}{k}$ with $\lim \gamma_n = \gamma$, the Euler-Mascheroni constant, compute the following:

- (i) $\lim_{n \rightarrow \infty} (\gamma_n - \gamma)n$;
- (ii) $\lim_{n \rightarrow \infty} (\gamma_n \gamma_{n+1} - \gamma^2)n$.

SOLUTIONS TO PROBLEMS 840 - 848

Problem 840. Proposed by the editor.

Consider the sequence $a_0 = 1, a_1 = 1, a_n = 2a_{n-1} + a_{n-2}$ which is 1, 1, 3, 7, 17, 41, 99, ...

a) Prove that no term in the sequence ends in 5.

[If you get this, send it.]

b) Prove that if p is prime, then $a_p \equiv 1 \pmod{p}$.

Solution by Brian Beasley, Presbyterian College, Clinton, SC.

a) For $n \geq 12$, applying the recurrence yields

$$a_n = 13860a_{n-11} + 5741a_{n-12}$$

and hence $a_n \equiv a_{n-12} \pmod{10}$. Thus the sequence cycles every 12 terms modulo 10. Since

$$\{a_n\}_{n=0}^{11} = \{1, 1, 3, 7, 7, 1, 9, 9, 7, 3, 3, 9\} \pmod{10},$$

we conclude that 5 never divides a_n .

b) We check directly that $a_2 = 3 \equiv 1 \pmod{2}$. Using the characteristic equation of the given recurrence, it is straightforward to verify that for every $n \geq 0$,

$$\frac{(1 + \sqrt{2})^n + (1 - \sqrt{2})^n}{2}.$$

For odd values of n , this in turn implies that

$$a_n = \sum_{i=0}^{(n-1)/2} \binom{n}{2i} 2^i.$$

Since p divides the binomial coefficient $\binom{p}{k}$ for $k \in \{1, 2, \dots, p-1\}$ whenever p is prime, we obtain

$$a_p = 1 + \binom{p}{2} 2^1 + \dots + \binom{p}{p-1} 2^{\frac{p-1}{2}} \equiv 1 \pmod{p}$$

for every odd prime p .

Part a) solved by Mohamed Lotfi (student), Wagner College, Staten Island, NY; and Ava Monson (student), North Central College, Naperville, IL. Both parts solved by Ethan Bamberger (student), North Central College, Naperville, IL; Michel Bataille, Rouen, France; Brian Bradie, Christopher Newport University, Newport News, VA; Cal Poly Pomona Problem Solving Group, Pomona, CA; Corneliu Mănescu-Avram, Ploiești, Romania; Missouri State University Problem Solving Group, Springfield, MO; Ioannis Sfikas, Athens, Greece; John Zerger, Catawba College, Salisbury, NC; Titu Zvonaru, Comănești, Romania; and the author.

Problem 841. Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.

Let a, b, c be the roots of the equation $x^3 - x^2 - 2x - 3 = 0$. Find the value of $a^5 + b^5 + c^5$.

Solution by Corneliu Mănescu-Avrân, Ploiești, Romania.

Denote $S_k = a^k + b^k + c^k$ for all positive integers k . From the Viète's relations, we obtain $S_1 = 1, S_2 = 1^2 - 2(-2) = 5$. Writing that a, b, c are roots of the given equation and adding the three equalities, we get $S_3 - S_2 - 2S_1 - 3 \cdot 3 = 0$, whence $S_3 = 16$. Multiplying the conditions that a, b, c are roots of the given equation by a, b, c , respectively and adding it, we get $S_4 - S_3 - 2S_2 - 3S_1 = 0$, whence $S_4 = 29$. Similarly, multiplying the conditions by a^2, b^2, c^2 , we get $S_5 - S_4 - 2S_3 - 3S_2 = 0$, whence $S_5 = 76$.

Also solved by Michel Bataille, Rouen, France; Brian Beasley, Presbyterian College, Clinton, SC; Brian Bradie, Christopher Newport University, Newport News, VA; Cal Poly Pomona Problem Solving Group, Pomona, CA; Ioan Viorel Codreanu, Satulung, Maramures, Romania; Saadat Hajiyeva (student), ADA University, Baku, Azerbaijan; Vien Ho (student), North Central College, Naperville, IL; Javid Khalilli (student), ADA University, Baku, Azerbaijan; Mohamed Lotfi, Wagner College, Staten Island, NY; Emin Rajabli (student), ADA University, Baku, Azerbaijan; Reagan Raymer (student), North Central College, Naperville, IL; Henry Ricardo, Westchester Area Math Circle, Purchase, NY; Ioannis Sfikas, Athens, Greece; John Zerger, Catawba College, Salisbury, NC; Michael Zirpoli (student), Slippery Rock University, Slippery Rock, PA; and the author.

Problem 842. Proposed by José Luis Díaz-Barrero, School of Civil Engineering, Barcelona Tech - UPC, Barcelona, Spain.

Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$4^{-(x+y)} \leq \frac{f(x)f(y)}{(x^4+1)(y^4+1)} \leq \frac{f(x+y)}{(x+y)^4+1}$$

for all $x, y \in \mathbb{R}$.

Solution by the Cal Poly Pomona Problem Solving Group, Pomona, CA.

Multiplying each side of the given inequality by $4^{(x+y)}$ yields

$$1 \leq \frac{4^x f(x) 4^y f(y)}{(x^4+1)(y^4+1)} \leq \frac{4^{(x+y)} f(x+y)}{(x+y)^4+1}.$$

Letting $h(x) = \frac{4^x f(x)}{x^4+1}$, the above becomes $1 \leq h(x)h(y) \leq h(x+y)$. Now letting $p(x) = \ln(h(x))$ and taking the natural logarithm of each side of the above yields

$$0 \leq p(x) + p(y) \leq p(x+y). \quad (1)$$

Making $x = y = 0$, we obtain $0 \leq 2p(0) \leq p(0)$. This inequality will only hold if $p(0) = 0$. Substituting $\frac{y}{2}$ for x and y in (1) yields $0 \leq 2p\left(\frac{y}{2}\right) \leq p(y)$. This shows that $p(y) \geq 0$ for all y . Since $p(y) \geq 0$, from (1) it follows that $p(x) \leq p(x+y)$ for all x and y . If we let $y > 0$, then $x < x+y$. Then p is increasing. If we let $y < 0$, then $x > x+y$. Then p is decreasing. Hence p is constant. Since p is constant and $p(0) = 0$, It must be true that $p(x) = 0$ for all x . Finally, $f(x) = \frac{x^4+1}{4^x}$.

Also solved by by Michel Bataille, Rouen, France; Mohamed Lotfi, Wagner College, Staten Island, NY; Ioannis Sfikas, Athens, Greece; Marian Ursărescu, National College "Roman Voda", Roman, Romania; Daniel Văcaru, Pitești, Romania; and the author.

Problem 843. *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.*

Prove that in $\triangle ABC$ you have

$$\sqrt{(2^{h_a} + 2^{h_b} + 2^{h_c})(2^{m_a} + 2^{m_b} + 2^{m_c})} < 2^a + 3^b + 4^c.$$

Solution *by Ioannis Sfikas, Athens, Greece.*

It is well-known that in every triangle: $h_s \leq m_a < \frac{b+c}{2}$. If we assume that function $f(x) = 2^x$, then $f'(x) = 2^x \ln 2 > 0$ and $f''(x) = 2^x (\ln 2)^2 > 0$. So, the function $f(x)$ is an increasing and convex function. Also, we have:

$$2^{h_a} \leq 2^{m_a} < 2^{\frac{b+c}{2}} \leq \frac{2^b + 2^c}{2},$$

and $2^{m_a} + 2^{m_b} + 2^{m_c} < 2^a + 2^b + 2^c$ and

$$\begin{aligned} \sqrt{(2^{h_a} + 2^{h_b} + 2^{h_c})(2^{m_a} + 2^{m_b} + 2^{m_c})} &\leq 2^{m_a} + 2^{m_b} + 2^{m_c} \\ &< 2^a + 2^b + 2^c \\ &< 2^a + 3^b + 4^c. \end{aligned}$$

Also solved by the author.

Problem 844. Proposed by Daniel Sitaru, “Theodor Costescu” National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.

Prove that if $0 < a < b < c < 1$, then

$$2 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a \ln a & b \ln b & c \ln c \end{vmatrix} > \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ (a-1) \ln(a^2+1) & (b-1) \ln(b^2+1) & (c-1) \ln(c^2+1) \end{vmatrix}.$$

Solution by Michel Bataille, Rouen, France.

Let $f : (0, 1) \rightarrow \mathbb{R}$. Then

$$\begin{aligned} \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ f(a) & f(b) & f(c) \end{vmatrix} &= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-b \\ f(a) & f(b)-f(a) & f(c)-f(a) \end{vmatrix} \\ &= (b-a)(c-b) \left(\frac{f(c)-f(b)}{c-b} - \frac{f(b)-f(a)}{b-a} \right). \end{aligned}$$

Applying this result first with $f(x) = 2x \ln x$ and then with $f(x) = (x-1) \ln(x^2+1)$ and observing that $(b-a)(c-b) > 0$, we obtain that the proposed inequality is equivalent to

$$\frac{g(c)-g(b)}{c-b} > \frac{g(b)-g(a)}{b-a} \quad (1)$$

where g denotes the function defined by $g(x) = 2x \ln x - (x-1) \ln(x^2+1)$. Now we calculate the first two derivatives of g :

$$\begin{aligned} g'(x) &= 2 + 2 \ln x - \ln(x^2+1) - \frac{2x^2-2x}{x^2+1} \\ g''(x) &= \frac{2(1-x^2)(1+x)}{x(x^2+1)^2}. \end{aligned}$$

We deduce that $g''(x)$ is positive when x is in $(0, 1)$. Thus g is convex on the interval $(0, 1)$ and (1) follows since $a < b < c$.

Also solved by Ioannis Sfikas, Athens, Greece; and the author.

Problem 845. Proposed by Daniel Sitaru, “Theodor Costescu” National Economic College, Drobeta Turnu – Severin, Mehedinti, Romania.

If $a, b, c \in [0, 1)$, then

$$8 \int_0^a \left(\int_0^b \left(\int_0^c \frac{\sin^{-1}x \cdot \sin^{-1}y \cdot \sin^{-1}z}{(1 + \sin^{-1}x)(1 + \sin^{-1}y)(1 + \sin^{-1}z)} dz \right) dy \right) dx \leq a^2 b^2 c^2.$$

Solution by the Missouri State University Problem Solving Group, Springfield, MO.

Let $t \in [0, 1)$. Consider the function $f(t) = t(1 + \arcsin t) - \arcsin t$. Since we have that

$$\frac{t-1}{\sqrt{1-t^2}} = -\frac{1-t}{\sqrt{(1-t)(1+t)}} = -\sqrt{\frac{1-t}{1+t}} \geq -1,$$

we see

$$f'(t) = 1 + \arcsin t + \frac{t}{\sqrt{1-t^2}} - \frac{1}{\sqrt{1-t^2}} \geq 0.$$

Since $f(0) = 0$, it follows that $t(1 + \arcsin t) \geq \arcsin t$ and so $\frac{\arcsin t}{1 + \arcsin t} \leq t$. We therefore have

$$\begin{aligned} & 8 \int_0^a \int_0^b \int_0^c \frac{\arcsin x \arcsin y \arcsin z}{(1 + \arcsin x)(1 + \arcsin y)(1 + \arcsin z)} dz dy dx \\ &= \int_0^a 2 \frac{\arcsin x}{1 + \arcsin x} dx \int_0^b 2 \frac{\arcsin y}{1 + \arcsin y} dy \int_0^c 2 \frac{\arcsin z}{1 + \arcsin z} dz \\ &\leq \int_0^a 2t dt \int_0^b 2t dt \int_0^c 2t dt \\ &= a^2 b^2 c^2. \end{aligned}$$

Also solved by Michel Bataille, Rouen, France; Brian Bradie, Christopher Newport University, Newport News, VA; Cal Poly Pomona Problem Solving Group, Pomona, CA; Soumava Chakraborty, “SoftWeb Technology”, Kolkata, India; Soumitra Mandal-Chanrar, Nagore, India; Ioannis Sfikas, Athens, Greece; and the author.

Problem 846. Proposed by Pedro H.O. Pantoja, Natal/RN, Brazil.

Evaluate $\int_0^{\pi/4} \cos^2(x) \cdot \ln(1 + \cos(4x))dx$.

Solution by Brian Bradie, Christopher Newport University, Newport News, VA.

Using the identities $1 + \cos 4x = 2\cos^2 2x$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$,

$$\begin{aligned} & \int_0^{\pi/4} \cos^2 x \ln(1 + \cos 4x) dx \\ &= \frac{\ln 2}{2} \int_0^{\pi/4} (1 + \cos 2x) dx + \int_0^{\pi/4} \ln(\cos 2x) dx + \int_0^{\pi/4} \cos 2x \ln(\cos 2x) dx \\ &= \frac{\ln 2}{8} (\pi + 2) + \frac{1}{2} \int_0^{\pi/2} \ln(\cos x) dx + \frac{1}{2} \int_0^{\pi/2} \cos x \ln(\cos x) dx. \end{aligned}$$

Now let $I = \int_0^{\pi/2} \ln(\cos x) dx$. Making the change of variables $x \rightarrow \frac{\pi}{2} - x$ yields $I = \int_0^{\pi/2} \ln(\sin x) dx$. And making the change of variables $x \rightarrow \pi - x$ in this integral yields $I = \int_{\pi/2}^{\pi} \ln(\sin x) dx$. Therefore

$$\begin{aligned} 2I &= \int_0^{\pi/2} \ln(\sin x) dx + \int_0^{\pi/2} \ln(\cos x) dx \\ &= \int_0^{\pi/2} \ln(\sin x \cos x) dx = \int_0^{\pi/2} \ln\left(\frac{1}{2} \sin 2x\right) dx \\ &= -\frac{\pi}{2} \ln 2 + \int_0^{\pi/2} \ln(\sin 2x) dx \\ &= -\frac{\pi}{2} \ln 2 + \frac{1}{2} \int_0^{\pi} \ln(\sin x) dx \\ &= -\frac{\pi}{2} \ln 2 + \frac{1}{2} \left(\int_0^{\pi/2} \ln(\sin x) dx + \int_{\pi/2}^{\pi} \ln(\sin x) dx \right) \\ &= -\frac{\pi}{2} \ln 2 + I, \end{aligned}$$

so $I = -\frac{\pi}{2} \ln 2$. With integration by parts

$$\begin{aligned} \int \cos x \ln(\cos x) dx &= \sin x \ln(\cos x) + \int (\sec x - \cos x) dx \\ &= \sin x \ln(\cos x) + \ln(\sec x + \tan x) - \sin x + C \\ &= \sin x (\ln \cos x) - \ln(\cos x) + \ln(1 + \sin x) - \sin x + C \\ &= (\sin x - 1) \ln(\cos x) + \ln(1 + \sin x) - \sin x + C. \end{aligned}$$

So

$$\int_0^{\pi/2} \cos x \ln(\cos x) dx = \ln 2 - 1 - \lim_{x \rightarrow \frac{\pi}{2}^-} (\sin x - 1) \ln(\cos x).$$

Using L' Hopital's rule twice, $\lim_{x \rightarrow \frac{\pi}{2}^-} (\sin x - 1) \ln(\cos x) = 0$ and so

$$\int_0^{\pi/2} \cos x \ln(\cos x) dx = \ln 2 - 1. \text{ Finally}$$

$$\begin{aligned} \int_0^{\pi/4} \cos^2 x \ln(1 + \cos 4x) dx &= \frac{\ln 2}{8} (\pi + 2) - \frac{\ln 2}{4} \pi + \frac{1}{2} \ln 2 - \frac{1}{2} \\ &= \frac{(6 - \pi) \ln 2 - 4}{8}. \end{aligned}$$

Also solved by Michel Bataille, Rouen, France; Kamel Benaicha, University Algiers, Algeria; Vien Ho (student), North Central College, Naperville, IL; Ioannis Sfikas, Athens, Greece; Remus Florin Stanca, Romania; Michael Zirpoli (student), Slippery Rock University, Slippery Rock, PA; and the author.

Problem 847. *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

Let $(x_n), (y_n)$ be positive sequences of real numbers such that $\lim_{n \rightarrow \infty} \frac{x_n}{n} = x$ and $\lim_{n \rightarrow \infty} (y_{n+1} - y_n) = y$. Evaluate $\lim_{n \rightarrow \infty} \left(\frac{y_{n+1}}{y_n} \right)^{x_n}$.

Solution by the authors.

We have that

$$\lim_{n \rightarrow \infty} \frac{y_n}{n} = \lim_{n \rightarrow \infty} \frac{y_{n+1} - y_n}{(n+1) - n} = \lim_{n \rightarrow \infty} (y_{n+1} - y_n) = y.$$

Then

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(\frac{y_{n+1}}{y_n} \right)^{x_n} &= \lim_{n \rightarrow \infty} \left(\left(1 + \frac{y_{n+1} - y_n}{y_n} \right)^{y_n / (y_{n+1} - y_n)} \right)^{x_n (y_{n+1} - y_n) / y_n} \\ &= e^z \end{aligned}$$

where

$$z = \lim_{n \rightarrow \infty} \frac{x_n}{n} * \frac{n(y_{n+1} - y_n)}{y_n} = x \frac{1}{y} = x,$$

if $x > 0$.

Also solved by Michel Bataille, Rouen, France; and Ioannis Sfikas, Athens, Greece pointing out with examples that y needs to be greater than 0 for the above answer.

Problem 848. *Proposed by D.M. Bătinetu-Giurgiu, “Matei Basarab” National College, Bucharest, Romania and Neculai Stanciu, “George Emil Palade” School, Buzău, Romania.*

If $m \in (1, \infty)$, n an integer greater than 1, a_k positive reals and

$\sum_{k=1}^n a_k = a$, then

$$\sum_{k=1}^n \left(\frac{a_k}{a_{k+1}} + \frac{1}{a_k} \right)^{1/m} \geq \left(\frac{2}{\sqrt{a}} \right)^{1/m} n^{1 + \frac{1}{2m}},$$

where $a_{n+1} = a_1$.

Solution by Henry Ricardo, Westchester Area Math Circle, Purchase, NY.

Using the AGM inequality first and then Radon's inequality, we have

$$\begin{aligned}
 \sum_{k=1}^n \left(\frac{a_k}{a_{k+1}} + \frac{1}{a_k} \right)^{1/m} &\geq \sum_{k=1}^n \left(\frac{2}{\sqrt{a_{k+1}}} \right)^{1/m} \\
 &= 2^{1/m} \sum_{k=1}^n \frac{1^{1+1/2m}}{(a_{k+1})^{1/2m}} \\
 &\geq 2^{1/m} \frac{(\sum_{k=1}^n 1)^{1+1/2m}}{(\sum_{k=1}^n a_{k+1})^{1/2m}} \\
 &= \frac{2^{1/m} n^{1+1/2m}}{a^{1/2m}} \\
 &= \left(\frac{2}{\sqrt{a}} \right)^{1/m} n^{1+1/2m}.
 \end{aligned}$$

Also solved by Michel Bataille, Rouen, France; Brian Bradie, Christopher Newport University, Newport News, VA; Ioan Viorel Codreanu, Satulung, Maramures, Romania; Mohamed Lotfi, Wagner College, Staten Island, NY; Ioannis Sfikas, Athens, Greece; Marian Ursărescu, National College "Roman Voda", Roman, Romania; and the proposers.

Citation for Dr. Rhonda McKee
The George R. Mach Distinguished Service
Award Recipient

April 12, 2019

The George R. Mach Distinguished Service Award is presented each biennium to an individual who has made major contributions to the Society. Nominations are solicited from the chapters and the National Council determines the recipient. The chapter with which the recipient is affiliated receives a monetary award of \$500.

I am pleased to announce that the recipient of the George R. Mach Distinguished Service Award for this biennium is Dr. Rhonda McKee.

Dr. McKee is the corresponding secretary of the MO Beta chapter at the University of Central Missouri. She has served for more than 33 years, starting in October of 1985. During that time she has initiated more than 550 students and faculty members. During her tenure, MO Beta has participated in all ten national conventions this century, often with one of the largest contingents in attendance.

Dr. McKee faithfully served at the national level for 18 years. She was the Director for the North Central Region from 1999 to 2001, National Secretary from 2000 to 2009, President-Elect from 2009 to 2013, and President from 2013 to 2017. More than 40 new KME chapters were installed while she served on the National Council.

Dr. McKee is also the program coordinator for mathematics at the University of Central Missouri. In 1999, she received the Distinguished College or University Teaching of Mathematics Award from the Missouri section of the Mathematical Association of America. She is currently serving as chair-elect for that section.

The KME National Council recognizes the dedication it takes to serve a single term on the council. Dr. McKee served more than four terms. We also realize how important it is for a chapter to have a corresponding secretary that is willing to organize travel for students to attend a single national convention. Dr. McKee has done this for decades. For this reason, we are very happy to present the George R. Mach Distinguished Service Award to Dr. Rhonda McKee.

Report of the 42nd Biennial National Convention

Kappa Mu Epsilon

April 11-13, 2019

Frostburg State University, Maryland Delta

Frostburg, Maryland

This 42nd Biennial National Convention, sponsored in part by the American Mathematical Society and American Statistical Association, was held during April 11-13, 2019 at Frostburg State University, Frostburg, Maryland, Host Chapter Maryland Delta.

Thursday, April 11, 2019

On Thursday evening, April 11th from 7-9 p.m. a Pizza/Ice Cream Reception and Mixer was held in Gira Center Room 397. Participants were able to pick up registration packets at this event. From 8-9 p.m., the National Council held a business meeting in Gira Center Room 245.

Friday, April 12, 2019

Friday April 12th's activities began at 8 a.m. to 9 a.m. with breakfast and continued registration in the Gira Center 1st floor lobby. From 8:30-8:55 a.m., members of the Awards Committee held a meeting in Gira Center Room 155. At 9 a.m. in Gira Center Room 397 the first general session began, with KME President Brian Hollenbeck presiding.

KME President Brian Hollenbeck and Dr. Kim Hixson, Frostburg State University Dean of the College of Liberal Arts and Sciences welcomed participants. Mark S. Hamner, KME Secretary, then called the roll. In attendance were 56 students and 30 faculty members, a total of 86 registrants, representing 18 chapters from eleven states (Alabama, Kansas, Maryland, Missouri, Nebraska, New York, North Carolina, Oklahoma, Pennsylvania, Texas, and West Virginia).

Chapters represented were Alabama Theta, Jacksonville State University; Kansas Alpha, Pittsburg State University; Kansas Beta, Emporia State University; Kansas Delta, Washburn University; Maryland Delta, Frostburg State University; Missouri Beta, University of Central Missouri; Missouri Theta, Evangel University; Nebraska Beta, University of Nebraska – Kearney; New York Lambda, Long Island University Post; New York Rho, Molloy College; North Carolina Zeta, Catawba College; Oklahoma Alpha, Northeastern State University; Pennsylvania Mu, Saint Francis University;

Pennsylvania Zeta, Indiana University of Pennsylvania; Pennsylvania Upsilon, Seton Hill University; Texas Gamma, Texas Woman's University; Texas Kappa, University of Mary Hardin-Baylor, and West Virginia Alpha, Bethany College.

Twelve talks, a 4-member recent graduate panel discussion, and five workshops (three which were presented twice) were on the program.

The following new chapter installed during the 2017-2019 biennium was recognized:

- Pennsylvania Upsilon, Seton Hill University, May 18, 2018

Leah Childers KME President-Elect of Kansas Alpha conducted the filing of delegates.

There was no old business.

Introduction of new business:

- Changes to the Constitution
- New candidates for Secretary
 - Nominees introduced: Steven Shattuck of Missouri Beta and Katherine Kime of Nebraska Beta

The following Student papers were presented during the morning Session #1, 9:30 a.m.-10:25 a.m.: Gira Center Room 397.

- *Multiplicative factorization in numerical semigroups*, by Matthew Enlow, Missouri Beta, Missouri State University
- *Intersections of Shortest Taxicab Paths in the Sierpinski Carpet*, by Rebekah Chase, Missouri Theta, Evangel University
- *An Analysis of EdReady and Class Attendance at Jacksonville State University*, by Joel Branham and Holly Sparkman, Alabama Theta, Jacksonville State University

Between 10:30 a.m. and 10:45 a.m. there was a refreshment break in the Gira Center 1st floor lobby.

The following Student papers were presented during the morning Session #2, 10:45 a.m.-11:55 a.m.: Gira Center Room 397.

- *Linear Regression in Pythagorean Football Wins*, by Tim Deyell, Pennsylvania Upsilon, Seton Hill University
- *The Algebra behind RSA Encryption*, by Sandy Hussain, New York Lambda, Long Island University
- *Googling Information Retrieval and SVD*, by Kayla Gill, New York Rho, Molloy College

- *A Comparison of Modeling Techniques for Behavioral Data*, by Marcus Shell, Alabama Theta, Jacksonville State University

Between 12 noon-12:15 p.m. a Group Photograph was taken in the Gira Center 1st floor lobby.

Lunch was held between 12:15 p.m.-2:00 p.m. in the Chesapeake Cafeteria. Concurrently the KME Resolutions Committee and the KME Auditing Committee met in the cafeteria.

From 2:00 p.m.-2:45 p.m., section meetings were held, The Faculty Section meeting was held in Gira Center Room 156. The Student Section meeting was held in Gira Center Room 397.

The following Student papers were presented during the afternoon Session #3, 2:45 p.m.-3:25 p.m.: Gira Center Room 397.

- *The Additive Property of the Sum-of-Divisors Function*, by Marcia-Mariel Erhart, North Carolina Zeta, Catawba College
- *Comparing Accuracy and Distance in the sport of Disc Golf*, by Katherine Beckley, Kansas Beta, Emporia State University

Between 3:30 p.m. and 5:00 p.m. participants attended two of five workshops (prior signup required):

- (Gira Center Room 222) Hull's Hybrid Unit Origami, by Marc Michael, Maryland Delta, Frostburg State University
- (Gira Center Room 245) Maya Geometry: Verification of a Right Angle, by John Diamantopoulos, Oklahoma Alpha, Northeastern State University and Cynthia Huffman, Kansas Alpha, Pittsburg State University
- (Gira Center Room 264) Exploring Game Mechanics in a Math or Stats Setting, by Gaspar Porta, Kansas Delta, Washburn University
- (Gira Center Room 223) What Mathematics Forrest Gump Can Teach You About Playing the Lottery, by Ron Wasserstein, American Statistical Association
- (Gira Center Room 223) Paths on Polyhedra, by John Snow, Texas Kappa, University of Mary Hardin-Baylor

Between 4:15 p.m. and 4:30 p.m. there was a refreshment break in the Gira Center 1st floor lobby.

From 7:00 p.m.-9:30 p.m. the convention banquet was held in the Alice R. Manicur Assembly Hall. Leah Childers, KME President-Elect, served as emcee. Following dinner, length of service awards were presented to

attending Corresponding Secretaries who had fulfilled 10 or more years of service. The numbers of secretaries around the nation who have served 10-14 years, 15-19 years, 20-24 years, etc. were also mentioned. Faculty sponsors were asked to stand and be recognized for their service. Brian Hollenbeck, KME President, then presented this year's Mach Award. This year's recipient of the Mach Award was Rhonda McKee, Missouri Beta. Dr. McKee is the corresponding secretary of the MO Beta chapter at the University of Central Missouri. She has served for more than 33 years, starting in October of 1985, during which time she has initiated more than 550 students and faculty members. Throughout her tenure, MO Beta has participated in all ten national conventions this century, often with one of the largest contingents in attendance. Dr. McKee faithfully served at the national level for 18 years. She was the Director for the North Central Region from 1999 to 2001, National Secretary from 2000 to 2009, President-Elect from 2009 to 2013, and President from 2013 to 2017. More than 40 new KME chapters were installed while she served on the National Council. Lastly, Ron Wasserstein, from the American Statistical Association, gave the keynote address: *Doctor, It Hurts When I p.*

Saturday, April 13, 2019

Saturday April 13th's activities began between 8:00 a.m. and 8:30 a.m. with Registration and breakfast in the Gira Center 1st floor lobby.

Session #4 of the Student Presentations commenced at 8:30 a.m. in Gira Center Room 397.

The following Student papers were presented during Session #4, 8:30 a.m.-9:25 a.m.:

- *We're skewed! Can we test our way out of it?* by Jacob Talkin, Kansas Delta, Washburn University
- *Elliptic Curve Cryptography* by Alissa Whiteley, Maryland Delta, Clarkson University (formerly Frostburg State University)
- *Depressed Cubic Curves with Intersecting Lines* by Ben Junkins, Alabama Theta, Jacksonville State University

Between 9:30 a.m. and 9:55 a.m. there was a refreshment break in the Gira Center 1st floor lobby. Concurrently, the KME Awards Committee met in Gira Center Room 245 and Regional meetings were held.

Between 10:15 a.m. and 10:40 a.m. a Recent Graduate Panel Discussion was held in Gira Center Room 397.

Panelists included:

- Michelle Welch, teacher at Gaithersburg High School

- Michael Shannon, technical consultant for IBM
- Alissa Whiteley, graduate student at Clarkson University
- Daniel Steel, graduate student at West Virginia University

Following at 11:00 a.m., also in Gira Center Room 397, the Second General Session began with President Brian Hollenbeck presiding. Convention Evaluation Forms were distributed and collected.

Elections were held for the positions of KME Secretary.

- Steven Shattuck was elected and installed

Mark S. Hamner of Texas Gamma completed his term as Secretary and Steven Shattuck was installed as Secretary by the President, Brian Hollenbeck. David Dempsey was installed as Treasurer for a second term. John Snow was installed as Webmaster which will be an elected position in the future.

The National Council presented an updated Constitution and Bylaws for consideration. All presented updates were approved unanimously by those in attendance. The updated version of the Constitution can be found on the society's website.

For the Continuation of New Business, the following national officers made reports:

- Doug Brown, Editor, The Pentagon
- John W. Snow, Webmaster
- Cynthia Huffman, Historian
- David Dempsey, Treasurer
- Mark S. Hamner, Secretary
- Leah Childers, President-Elect
- Brian Hollenbeck, President

Following the national officer reports were reports from the Section Meetings, the Resolutions Committee, and the Auditing Committee. All reports are attached.

Mark Hamner
National Secretary

Report of the National President

It has been two years since I was installed as president of KME. It has been a pleasure working with my fantastic colleagues on the national council. Here are some highlights of what we have been working on:

- Several boxes of historical minutes, correspondence, and other relics of the society were scanned for electronic storage. These files are now stored in an online archive using Google Drive. The original documents were shipped to the Archives of American Mathematics at the University of Texas at Austin.
- KME pens were produced and distributed to chapters.
- The Mach Award stipend was increased to \$500.
- In the summer of 2018, John Snow of the Texas Kappa chapter was appointed as Webmaster and Vanessa Williams of the Missouri Eta chapter was appointed to be Social Media Coordinator. Outgoing Webmaster, David Gardner, was honored via Skype at the North Central/South Central regional convention in April of 2018.
- Chapter documents have been updated. This includes revising the Constitution which is being considered for approval at this convention.
- We've started a plan to recognize corresponding secretaries who have served 10, 15, 20, ... years.
- We installed three new chapters this biennium:
 - Kansas Eta chapter at Sterling College in Sterling. Installed by Brian Hollenbeck on Nov. 30, 2017.
 - New York Sigma chapter at the College of Mount Saint Vincent in The Bronx. Installed by Brian Hollenbeck on April 30, 2018.
 - Pennsylvania Upsilon chapter at Seton Hill University in Greensburg. Installed by Pete Skoner on May 18, 2018.

I also want to recognize our excellent regional directors, who organized four regional conventions in 2018:

- South Eastern: April 6-7, 2018 at Georgia Gwinnett College in Lawrenceville, Georgia.
- New England: April 7, 2018, at St. Joseph's College in Patchogue, New York.
- Great Lakes: April 7, 2018 at Wheeling Jesuit University in Wheeling,

West Virginia. Co-hosted with Bethany College.

- North Central/South Central: April 13-14, 2018 at Emporia State University in Emporia, Kansas.

I believe this is a record number of regional conventions, at least for recent years.

I encourage everyone to check out the most recent issue of *The Pentagon*. This is the fifth issue to be published by Editor Doug Brown. I appreciate his efforts, along with Problem Corner Editor, Pat Costello, and the referees.

I would also like to extend special thanks to President-Elect Leah Childers, Mark Hughes, and the Maryland Delta chapter for hosting this convention. I know it is a lot of hard work to organize a national convention.

Finally, I wish outgoing Secretary, Mark Hamner, all the best. I have really enjoyed working with him the past six years I've been on the council. With over 150 KME chapters, the Secretary has to juggle a lot of information, and Mark has done an outstanding job.

Brian Hollenbeck
National President

Report of the National President-Elect

This is my second year as President-Elect. It continues to be a pleasure working with the rest of the National Council. In particular, I would like to wish Mark Hamner the best as his term as secretary is coming to an end. I look forward to the next secretary continuing his tradition of excellence.

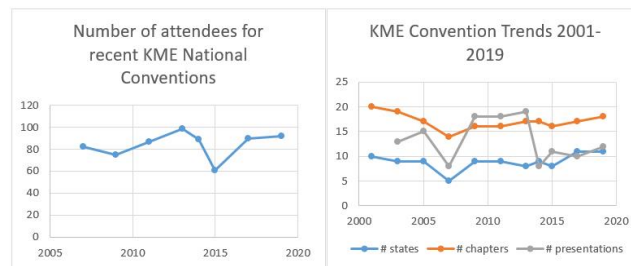
2019 National Convention

Kappa Mu Epsilon's 42nd Biennial Convention, which is KME's 43rd national convention overall, is being held this weekend, April 11-13 in Frostburg, Maryland. Our host chapter is Maryland Delta at Frostburg State University. There are 18 chapters in attendance from 11 states (Alabama, Kansas, Maryland, Missouri, Nebraska, North Carolina, Oklahoma, Pennsylvania, Tennessee, Texas, and West Virginia). 12 presentations will be given by students from these chapters. There will also be a panel discussion and five workshops given over the course of the next two days. There are 92 KME members registered for the convention.

Once again, the AMS and the ASA have contributed \$500 each, which will be used to help defray the cost of student travel to the convention. We are certainly grateful for their support.

Finally, I would like to extend special thanks to Mark Hughes and the Maryland Delta chapter for hosting this convention. Their efforts will make this convention a successful and enriching experience.

These charts show how this year's convention compares to other years.



In the 2000's, 58 different chapters have participated. Five have participated in all ten conventions. Four chapters are participating for the first time this century.

Following the tradition of recent President-Elects, here are some statistics about recent conferences.

In 2017 the host chapter was Missouri Theta at Evangel University. There are 17 chapters in attendance from 11 states (Alabama, Georgia, Kansas, Missouri, Nebraska, North Carolina, Oklahoma, Pennsylvania, Tennessee, Texas, and Virginia). 10 presentations were given by students

from these chapters. There was also be a panel discussion and four workshops given. Ninety KME members of KME attended.

In 2015 the host chapter was Florida Delta at Embry-Riddle Aeronautical University. There were 16 chapters in attendance from eight states (Alabama, Florida, Kansas, Michigan, Missouri, Pennsylvania, Texas, and West Virginia). 11 presentations were given by students from these chapters. In a departure from tradition, we also had four Florida universities join us as part of the Central Florida Undergraduate Mathematics Conference. Their participation allowed us to feature two parallel sessions, which was a first for KME conventions of recent memory. Another first was an integral bee. Sixty-one members of KME attended.

In 2014 the host chapter was Alabama Theta at Jacksonville State University. There were 17 chapters in attendance from nine states (Alabama, Kansas, Michigan, Missouri, Pennsylvania, Rhode Island, South Carolina, Texas, and West Virginia). Eight talks and a panel discussion were given as well as three workshops, which was a new feature of the KME convention. This was also the first time that KME held a national convention during an even-numbered year. Eighty-nine people attended the convention.

In 2013, the host chapter was Kansas Delta at Washburn University and there were 17 chapters in attendance from eight states (Alabama, Kansas, Louisiana, Michigan, Missouri, New York, Pennsylvania, and Texas). Nineteen talks and a panel discussion were given over the course of two days. Ninety-nine people attended.

In 2011, 16 chapters from nine states (Indiana, Kansas, Kentucky, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in the convention in St. Louis, Missouri. Eighteen papers were presented. Eighty-seven people attended.

In 2009, 16 chapters from nine states (Georgia, Kansas, Maryland, Michigan, Missouri, New York, Oklahoma, Pennsylvania, and Texas) participated in Philadelphia, PA. Sixteen students presented papers. Seventy-five people attended.

In 2007, 14 chapters from five states (Kansas, Missouri, New York, Oklahoma, and Tennessee) participated in Springfield, Missouri. Eight students presented papers. 82 people attended.

In 2005 (Schreiner U., Kerrville, TX), there were 17 chapters from nine states (California, Kansas, Missouri, Michigan, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). There were 15 student presentations.

In 2003 (ORU, Tulsa, OK), there were 19 chapters from 9 states (Iowa, Kansas, Michigan, Missouri, New York, Oklahoma, Pennsylvania, Tennessee, and Texas). Thirteen student papers were presented.

In 2001 (Washburn U., Topeka, KS), there were 20 chapters from 10 states (Colorado, Iowa, Kansas, Kentucky, Missouri, New York, Oklahoma, Ohio, Pennsylvania, and Tennessee)

The following chapters have participated in at least one of the last ten conventions:

noindent Alabama Alpha (2014)
Alabama Beta (2014)
Alabama Epsilon (2014)
Alabama Zeta (2014)
Alabama Theta (2013, 2014, 2015, 2017, 2019)
California Epsilon (2005)
Colorado Delta (2001)
Georgia Alpha (2009)
Georgia Zeta (2017)
Florida Delta (2015)
Indiana Delta (2011)
Iowa Alpha (2001, 2003)
Iowa Gamma (2001)
Kansas Alpha (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015, 2017, 2019)
Kansas Beta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015, 2017, 2019)
Kansas Gamma (2001, 2003, 2007, 2015)
Kansas Delta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015, 2017, 2019)
Kansas Epsilon (2001)
Kentucky Alpha (2001, 2011)
Louisiana Delta (2013)
Maryland Beta (2009)
Maryland Delta (2019)
Maryland Epsilon (2009)
Michigan Beta (2003, 2005, 2009, 2011, 2013, 2014, 2015)
Missouri Alpha (2001, 2003, 2007, 2013, 2015, 2017)
Missouri Beta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015, 2017, 2019)
Missouri Eta (2017)
Missouri Theta (2001, 2003, 2005, 2007, 2009, 2011, 2013, 2014, 2015, 2017, 2019)
Missouri Iota (2001, 2003, 2005, 2007, 2009, 2011, 2013)
Missouri Kappa (2001, 2003, 2005, 2007)
Missouri Lambda (2013, 2015)

Missouri Mu (2011)
Nebraska Beta (2017, 2019)
New York Eta (2001, 2003, 2005, 2007, 2013)
New York Lambda (2003, 2005, 2019)
New York Omicron (2005, 2009, 2011, 2013)
New York Rho (2011, 2013, 2019)
North Carolina Zeta (2017, 2019)
Ohio Alpha (2001)
Oklahoma Alpha (2003, 2007, 2017, 2019)
Oklahoma Gamma (2001, 2003, 2007)
Oklahoma Delta (2001, 2003, 2005, 2007, 2009, 2011)
Oklahoma Epsilon (2017)
Pennsylvania Theta (2001)
Pennsylvania Lambda (2003, 2009)
Pennsylvania Mu (2005, 2009, 2011, 2013, 2014, 2015, 2017, 2019)
Pennsylvania Tau (2014, 2015)
Pennsylvania Upsilon (2019)
Pennsylvania Zeta (2019)
Rhode Island Beta (2014)
South Carolina Delta (2014)
Tennessee Gamma (2001, 2003, 2005, 2007, 2017)
Texas Alpha (2013, 2015)
Texas Gamma (2003, 2005, 2009, 2011, 2013, 2014, 2015, 2017, 2019)
Texas Kappa (2019)
Texas Mu (2003, 2005, 2009, 2011)
Virginia Alpha (2017)
West Virginia Alpha (2014, 2015, 2019)

Leah Childers
President-Elect

Report of the National Secretary

Kappa Mu Epsilon, National Mathematics Honor Society initiated 2,232 new members in 129 chapters during the 42nd Biennium that ended March 15, 2019. That brings the total membership of KME to 88,260. Thirty-one active chapters did not report any initiates during the 42nd biennium.

As National Secretary, I receive all initiation reports from chapters, make a record of those reports, up-date mailing list information for corresponding secretaries and forward copies of the reports to other officers. At the beginning of each new biennium, I prepare a new KME brochure. During an academic year, I send out supplies to each chapter. The supplies include information brochures, membership cards and one or two copies of the brochure "A Matter of Honor." As a token of our appreciation from the National Council, Fall 2018 supplies included a small gift (KME pens) of thanks to our corresponding secretaries and organization. When a college or university petitions for a new chapter of KME, I send out a summary of the petition, prepared by the president, to each chapter and receive the chapter ballots.

My tenure on the KME National Council will come to an end at this convention. I have served as the National Secretary since 2009 after Rhonda McKee vacated the position to serve President-Elect. Serving on this Council has been an incredible honor and rewarding experience mostly because of the people I have been privileged to work on this council. I want to note my deep appreciation to the 2009 council members Ron Wasserstein, Rhonda McKee, Cynthia Huffman, and Pete Skoner for welcoming me to serve on the KME National Council. I am grateful to have served with the incredible members, Brian Hollenbeck, David Dempsey, Leah Childers, Cynthia Huffman, and John Snow. It has been an absolute pleasure to serve with all of you and I am thankful for your friendship.

Mark Hamner
National Secretary

Report of the National Historian

I am pleased to submit this report at the end of my first biennium of service as the National Historian of the Kappa Mu Epsilon National Mathematics Honor Society. Thanks are owed to Dr. Pete Skoner, past KME historian, for his help in the transition. It has been a pleasure to serve with the national and regional officers, with the corresponding secretaries and faculty sponsors of individual chapters, and with students during the last two years. Kappa Mu Epsilon is a great organization which has touched the lives of many people.

The primary function of the national historian is to solicit, collect, maintain and compile records of chapter activities, installation of new chapters, and other society activities of historical significance. Chapter News reports for fall and spring semester are solicited from corresponding secretaries via email. This regular messaging provides the opportunity to learn about chapter leadership changes and to maintain communication between the national society and the local chapters, hopefully helping to fulfill the mission of the organization to recognize students who excel in mathematics. The work of the National Historian is impossible without the aid of the corresponding secretaries for each chapter. Thank you for all that you do in serving the students at your institution, your local chapter, and the national organization!

Thank you all for your service, commitment, communication, and friendship. The passion and time all of you give contributes to this great organization that will celebrate our 88th anniversary since our founding on April 18, 1931, as we gather for our 43rd national convention. This 43rd national convention happens to be our 14th prime-numbered convention, our eleventh twin-prime-number convention, and the first prime-numbered convention formed from reverse concatenation of two consecutive numbers. (Just to clarify, this would have been our 44th national convention in 88 years, but two were not held during WWII, and then for the first time in 2014 we held a convention in an even-numbered year.)

During the past biennium from March 2017 to March 2019, 58 different chapters responded at least once to the chapter news request, a decrease from 67 in the previous biennium. Special mention goes to the following 19 chapters (up from 16 last biennium) for their cooperation in responding to all four inquiries: AL Theta, IA Alpha, IL Zeta, KS Alpha, KS Beta, MD Alpha, MD Delta, MI Delta, MO Beta, MO Theta, MS Gamma, NE Beta, NY Omicron, OH Theta, PA Mu, PA Rho, RI Beta, TN Gamma, and VA Beta.

The 90th Anniversary of KME is coming up in 2021. In honor of that

anniversary, I am at work updating the 82nd Anniversary KME History and Information Booklet that was distributed at the 39th national convention at Washburn University into a 90th Anniversary KME History and Information Booklet.

A special thank you goes to the editor of *The Pentagon* during this biennium, Doug Brown of the North Carolina Zeta Chapter. The edited Chapter News section is sent to the editor after each semester, and Doug has been amazing to work with.

Work is in progress to move records to a Google Drive archive. A big thank you to Dr. Pete Skoner, past KME historian, for beginning the process of putting documents, including chapter records, on the Google Drive.

Thank you all for allowing me to serve this wonderful organization!

Cynthia J. Huffman
National Historian

Report of the National Treasurer

42nd Biennium (March 16, 2017 – March 15, 2019)

A Biennium Asset Report and Biennium Cash Flow Report are given below. The Asset Report shows biennium assets of **\$106,960.56**. The Cash Flow Report shows that we have an **asset gain of \$1,779.60** this biennium.

BIENNIUM ASSET REPORT

Total Assets at beginning of 41st Biennium (March 16, 2017) \$105,180.96

Current Assets (Wells Fargo Bank)	
Checking	28,158.44
Savings	40,155.19
Time Account xxx7120	10,060.18
Time Account xxx7138	10,090.40
Time Account xxx7146	10,151.12
Time Account xxx7153	10,151.12
Total Current Assets (as of March 31, 2019)	\$108,766.45
- uncleared check #1642	- \$62.68
Total Current Register Assets (as of March 31, 2019)	\$108,703.77
- Activity since end of 42nd Biennium (March 15, 2019) - \$ 1743.21	
Total Assets (Register) at end of 42nd Biennium	\$106,960.56

BIENNIUM CASH FLOW REPORT

Receipts	
Initiation fees received	44,355.00
Installation fees received	150.00
Interest income	400.38
Gifts & misc. income	1595.58
Total Biennium Receipts	\$46,500.96
Time Account (x7120)	
renewal – deposit	10,043.43
Total Biennium Inflow	\$ 56,544.39

Expenditures	
Association of College Honor Societies	1,308.00
Administrative expenses	6,182.70
National Convention expenses	13,664.62
Regional Convention expenses	5,697.05
Council Meetings	2,599.63
Certificates, jewelry & shipping	13,504.31
Installation expenses	711.71
Miscellaneous (overpayment)	1,053.34
Total Actual Biennium Expenses	\$44,721.36
Time Account (x7120) renewal—deposit	10,043.43
Total Biennium Outflow	\$ 54,764.79
Biennium Cash Flow	\$ 1,779.60

The cash flow last biennium (2015-17) was **\$9,132.60**. The National Council passed on part of this increase to the students and chapters by giving an extra amount for the 2017 National Convention (\$200 above normal allocations) and also giving travel allocations for the 2018 Regional Conventions (Reg. 1—New England, Reg. 3—South Eastern, & Reg. 4/5—North/South Central: \$200 to each chapter attending). Each chapter is also receiving an extra amount in their travel allocation for the 2019 National Convention (\$200), which will show up in the next biennium report. We have easily continued to meet our National Council goal of maintaining assets of at least \$40,000. The financial condition of Kappa Mu Epsilon is sound. We have added additional assurance by maintaining a fidelity bond insuring Kappa Mu Epsilon, Inc., against any losses resulting from dishonesty of the 5 main officers (however unlikely). As I wrap up my first term as national treasurer, I want to thank my colleagues on the National Council for their untiring dedication, as well as the corresponding secretaries who maintain such a vital role in Kappa Mu Epsilon. I am grateful for the opportunity to serve with such outstanding individuals in encouraging and recognizing students for accomplishments in mathematics.

David Dempsey
KME National Treasurer

Report of *The Pentagon* Editor

Introduced in 1941, *The Pentagon*, is the official publication of Kappa Mu Epsilon. Publication of student papers continues to be the focus of *The Pentagon*. Following tradition, papers given “top” status and other recognition by the Awards Committee at the KME National Convention are guaranteed an opportunity to be published. *The Pentagon* is now completely electronic and available for free online via the KME website: www.kappamuepsilon.org.

I have been the Editor of *The Pentagon* since November 2016. The typical duties involve corresponding with authors of potential articles for submission, and facilitating referee feedback and author corrections for upcoming issues. Since the last national convention, four issues of *The Pentagon* (Fall 2016, Spring 2017, Fall 2017 and Spring 2018) have been published and made available on the website. The Fall 2018 issue is near completion and my goal is to have Spring 2019 published prior to the end of the summer. The Problem Corner continues to be made available on the KME website ahead of publication of the full issue.

A regular publication schedule requires a steady stream of articles from which to select, so assistance from the chapters in encouraging students to submit papers will be greatly appreciated.

The publication of *The Pentagon* would not be possible without the dedication of the referees, whose thoughtful reviews have been invaluable in helping authors fine-tune their submissions.

Stephen Andrilli	LaSalle University
Peter Chen	The University of Mary Hardin–Baylor
Chip Curtis	Missouri Southern State Univ.
Tara Davis	Hawaii Pacific University
Vincent Ferlini	Keene State College
Adam Fletcher	Bethany College
Tim Flood	Pittsburg State Univeristy
Thomas Kent	Marywood University
Tom McNamara	Southwestern OK State Univ.
J. Lyn Miller	Slippery Rock University
Lloyd Moya	Henderson State University
Ann Podleski	Harris-Stowe State Univ.
Sara Quinn	Dominican University
Adam Salminen	University of Evansville
Manyiu Tse	Molloy College
John Zerger	Catawba College

Finally, I am very grateful to the Associate Editors: Pat Costello, who

organizes the Problem Corner for each issue, and KME Historian Cynthia Huffman, who collects and prepares the KME News Items, as well as KME Webmaster John Snow. Their patience and attention to detail are very much appreciated.

Doug Brown
Editor, *The Pentagon*

Report of the Audit Committee

Audit Committee Members:

Dianne Twigger, Missouri Theta, faculty, chair
Amber Innes, Kansas Beta, student
Katherine Kime, Nebraska Beta, faculty
Demetrick McDonald, Maryland Delta, student
Rahasya Bharaniah, Kansas Delta, student
John Harris, Pennsylvania Mu, faculty

Audit Process

1. Prior to the national convention, Treasurer David Dempsey provided electronic copies of the biennium financial summary data to the committee chair to facilitate verification of asset account totals prior to the convention. The Audit Committee Chair subsequently contacted Wells Fargo Bank (Jacksonville, Alabama) by telephone. The account balances for the Kappa Mu Epsilon Platinum Business Checking account, Platinum Business Savings accounts, and the four Time accounts were verified to correspond to the associated totals found on Treasurer Dempsey's biennium reports and current records. This verification was conducted on April 8, 2019 for the balances as submitted by Treasurer Dempsey on April 7, 2019.
2. At the convention, Treasurer Dempsey provided the committee chair with detailed documentation for receipt and payment transactions, monthly bank account statements and reconciliation documentation, expense reports, receipts, income information, as well as his own reports and summary for the full biennium. The chair shared these documents with the committee for review.
3. At the national convention, the Audit Committee Chair interviewed President Brian Hollenbeck and Secretary Mark S. Hamner to determine their impressions of the accuracy and completeness of the recording of the financial transactions throughout the biennium.

Findings

1. The bank information provided by Treasurer Dempsey (as of March 15, 2019) was verified by the committee chair on April 8, 2019 via a phone conversation with Mrs. Donna Kopet, of Wells Fargo Bank (Jacksonville, Alabama).
2. The committee spot checked the Secretary's report and corresponding

computer generated Treasurer's report and found no inconsistencies throughout. We talked to the Secretary who reported perfect compliance by Treasurer Dempsey on this subject.

3. The committee spot checked the expense payment reports and receipts provided and found no inconsistencies. The President expressed total satisfaction with the integrity of the process.
4. The committee inspected monthly financial institution statements, quarterly interest statements, and CD interest reports. We compared those to the Treasurer quarterly reports. We found complete consistency between bank statements throughout the biennium.

Recommendations

1. Information forwarded by the Treasurer to the committee chair prior to the national convention provides the opportunity for verification of assets in a careful and timely manner. This should be continued for future audit committees.
2. The organization files an electronic tax notecard annually even though no taxes are required. The committee recommends that this practice continue.
3. The internal checks built into the regular financial processing between the Treasurer, President, and Secretary provide an important safeguard to the integrity of the office of the Treasurer and help avoid the necessity of an expensive external audit. These ongoing internal audit processes should be continued and updated by the National Council as needed.
4. The committee recommends that for future audit committees, a copy of approved policies and safeguards be provided electronically to the audit committee chair with biennium financial summary data.

Commendations

1. The committee commends Treasurer David Dempsey for his exemplary maintenance, management, and presentation of the financial records, and are grateful for his continued appointment as treasurer.
2. We further commend him for his valuable input and transparency throughout the process. His detailed written guidelines were extremely helpful for the Audit Committee.
3. The committee commends the national President, Secretary, and Treasurer for the manner in which they communicate and cooperate to maintain the internal checks that preserve the integrity of the office of Tre-

sure.

4. The committee commends the work of the previous audit committees, and is thankful for the sample reports provided by the Treasurer.

Dianne Twigger
Chair, Audit Committee

Report of the Resolutions Committee

The Resolutions Committee consisted of:

- Holly Sparkman (student member from Alabama Theta chapter),
- Gaspar Porta (faculty member from Kansas Delta chapter),
- Derek Noe (student member from Missouri Beta chapter),
- Jared Burns (faculty member from Pennsylvania Upsilon chapter), and
- Adam Fletcher (faculty member from West Virginia Alpha chapter).

This Committee hereby proposes the following resolutions.

“Whereas the success of any undertaking relies heavily upon the dedication and ability of its leaders, be it resolved:

1. That this Forty-Second Biennial National Convention express its gratitude
 - a. to Brian Hollenbeck (national president), Leah Childers (president-elect), Mark Hamner (national secretary), David Dempsey (national treasurer), Cynthia Huffman (national historian), and John Snow (national webmaster);
 - b. to Mark Hamner for ten years of hard work and dedication to the office of national secretary;
 - c. to Doug Brown for his service as editor of *The Pentagon*; and
 - d. to Donna Marie Pirich, Pete Skoner, Jamye Curry, Katherine Kime, and John Diamantopoulos for their service as regional directors.
 - e. That this Convention acknowledge the participation of the students and faculty who served on the Auditing, Awards, Nominating, and Resolutions committees, which is so essential for the success of the meeting.

“Whereas the primary purpose of Kappa Mu Epsilon is to encourage participation in mathematics and the development of an appreciation for its beauty, be it further resolved:

1. 1. That students Matthew Enlow, Rebekah Chase, Joel Branham, Holly Sparkman, Tim Deyell, Sandy Hussain, Kayla Gill, Marcus Shell, Marcia-Mariel Erhart, Katherine Beckley, Jacob Talkin, Alissa Whitely, and Ben Junkins, who prepared, submitted, and then presented their papers be given special commendation by this Forty-Second Biennial Convention for their enthusiasm and dedication, and

2. That this Convention express its thanks
 - a. to career panelists Marc Michael, Michelle Welch, Michael Shannon, Alissa Whiteley, and Daniel Steel;
 - b. to workshop hosts Marc Michael, John Diamantopoulos, Cynthia Huffman, Gaspar Porta, Ron Wasserstein, and John Snow; and
 - c. to Ron Wasserstein for his outstanding keynote address “Doctor, It Hurts When I ‘p’,” at the Friday night banquet.

“Finally, whereas Frostburg State University and the surrounding community of Frostburg, Maryland has provided this Convention with gracious hospitality, be it resolved:

1. That this Forty-Second Biennial Convention express its heartfelt appreciation to the Maryland Delta chapter for the thorough arrangements they have planned and carried out so successfully, and
2. to the American Mathematical Society and to the American Statistical Association, through whose grant support the operations of this Convention were augmented;
3. That this Convention recognize and thank Dr. Kim Hixon, Dean of the College of Liberal Arts and Sciences at Frostburg State University, as well as Dr. Mark Hughes, together with all the other members of Maryland Delta, who devoted their time and talents to ensure the success of this meeting.”

These resolutions respectfully submitted,
Adam C. Fletcher, Chair

Report of the Awards Committee

The Awards Committee met to select the three award winners. These are:

- 1-1 “Multiplicative factorization in numerical semigroups”, Matthew Enlow, Missouri Beta, University of Central Missouri
- 3-1 “The Additive Property of the Sum-of-Divisors Function”, Marcia-Mariel Erhart, North Carolina Zeta, Catawba College
- 4-3 “Depressed Cubic Curves with Intersecting Lines”, Ben Junkins, Alabama Theta, Jacksonville State University

People’s Choice:

1-2 “Intersections of Shortest Taxicab Paths in the Sierpinski Carpet”, Rebekah Chase, Missouri Theta, Evangel University

Respectfully submitted.
Leah Childers, Chair

Faculty Sectional Meeting

President Brian Hollenbeck opened up the meeting about the purpose of section meetings, which is to share ideas.

Participants introduced themselves.

A brief announcement was made on Problems Corner submissions for *The Pentagon*.

Discussion ensued on the following topics:

- Tax ID issue
 - National is 501C (tax exempt)
 - Most Honor Societies have subsidiaries file tax ID numbers, we do not
 - Questions: Do you have tax ID number? How do you handle finances? Issues?
 - * MS Theta uses university account
 - * Some have accounts before tax ID number
 - * Employer ID number (EIN)
 - Does not seem to be any issues with the way it is handled now
- Interest in having presence at national meetings (JMM, MathFest, MAA)
 - Pi Mu Epsilon relation with KME?
 - Booth at joint meetings
 - Membership: KME vs Pi Mu Epsilon
 - * Anything written about differences of Pi Mu Epsilon vs KME (traditionally small schools)?
- Website Suggestions
 - John wants to make website more mobile friendly
 - Can we get a directory of some oft to keep in touch (contacts)
 - * Corresponding Secretary list online:
 - Problems: Phishing, having to keep updated.
 - * Having a group picture of Faculty that you can tag (like Facebook)

- Infinity Award
 - Pi Mu Epsilon does this
 - One chapter wins that exhibits strength
 - Rubric for us to decide?

The session ended with thanking the faculty participants and Corresponding Secretaries.

Kappa Mu Epsilon News

Edited by Cynthia Huffman, Historian

Updated information as of June 2020

News of chapter activities and other noteworthy KME events should be sent to

Cynthia Huffman, KME Historian

Pittsburg State University

Mathematics Department

1701 S. Broadway

Pittsburg, KS 66762

or to

cjhuffman@pittstate.edu

KAPPA MU EPSILON

Installation Report

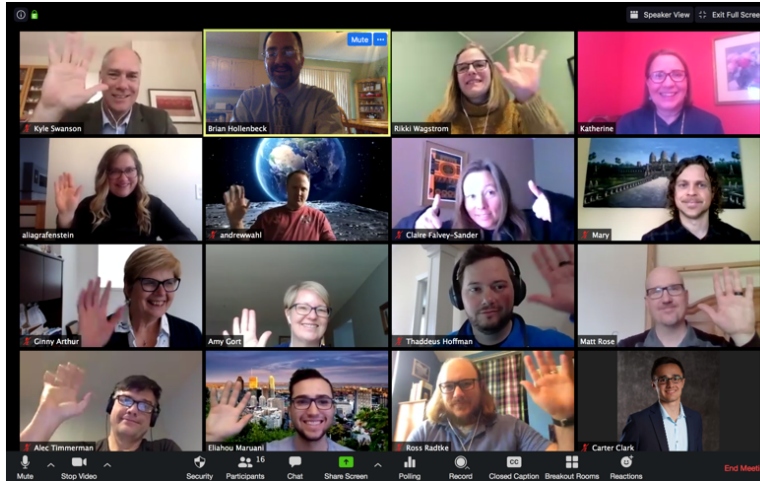
Minnesota Alpha, Metropolitan State University

Saint Paul Minnesota

The Minnesota Alpha Chapter of Kappa Mu Epsilon was installed at 10:00 a.m. on April 20, 2020 at a ceremony held via Zoom. Metropolitan State is the first university in Minnesota to host a chapter of KME, and also the first ever to be installed via Zoom. The meeting was conducted by the Dean of the College of Sciences, Dr. Kyle Swanson. KME national president, Dr. Brian Hollenbeck, served as the installing officer.

Fourteen students and three faculty members were initiated as the charter members of the Minnesota Alpha Chapter. The three faculty are Professor of Mathematics Dr. Cindy Kaus, and Associate Professors of Mathematics, Ms. Katherine Johnson and Dr. Rikki Wagstrom. The fourteen students are Matthew Ambers, Carter Clark, Jason Evans, Claire Falvey-Sander, Alia Grafenstein, Sarah Hartzel, Thaddeus Hoffman, Joshua Illg, Eliahou Maruani, Ross Radtke, Matthew Rose, Brian Strand, Alec Timmerman, and Andrew Wahl. The first officers of the chapter were installed: Eliahou Maruani, President; Alia Grafenstein, Vice President; Matthew Rose, Secretary; Thaddeus Hoffman, Treasurer; Rikki Wagstrom, Corresponding Secretary; and Katherine Johnson, faculty sponsor.

Following the installation ceremony, President Ginny Arthur, Provost Amy Gort, and Dean Kyle Swanson delivered congratulatory remarks to the charter members. The event concluded with Dr. Hollenbeck presenting a talk titled, *Mathematics vs. Gerrymandering*.



Minnesota Alpha

Chapter News

AL Alpha – Athens State University

Corresponding Secretary – Dr. Patricia Glaze; 309 Total Members; 4 New Members

New Initiates –Emi Isabella Dutton, Lillian Mae Hardman, Carlee Anne Landers, and Amanda Rhea Rivera.

AL Zeta – Birmingham-Southern College

Corresponding Secretary – Dr. Allie Ray; 618 Total Members; 7 New Members

New Initiates – Juan Campos, Emma Choate, Ryan Emili, Brett Martin, Emilee Olsen, Tristan Sumpter, and Noah Young.

AL Theta – Jacksonville State University

Chapter President – Marcus Shell; 289 Total Members; 13 New Members
Other Spring 2020 Officers: Benjamin Junkins, Vice President; Sabin Banjara, Secretary; Leanne Powell, Treasurer; and Dr. David Dempsey, Corresponding Secretary and Faculty Sponsor

The Alabama Theta Chapter met biweekly (including several pizza/game nights) during Spring 2020 until our campus was closed on March 13 due to the Covid-19 pandemic. Unfortunately, this was exactly the date of our annual initiation ceremony, so we were unable to initiate our 13 new members as planned (and the ice cream for the social afterward is still in our department freezer). Before this date, we did manage to finalize three T-shirt designs (one specifically for KME and two for our Math Club in

general), collect order payments, and place the order, but we have been unable to distribute the T-shirts thus far. We are planning to devise a distribution strategy once some campus restrictions have lifted, and we intend to hold officer elections when we return in the fall. We will be looking for ways to keep students connected and engaged, regardless of what our “new normal” looks like.

New Initiates – Taylor Anne Beckham, Hannah Reilly Boozer, Matthew Joseph Chiaravalloti, Aubrey Guidry, Nicholas Will Harper, Catherine Jenkins, Christian Chad Leonard, Savannah Rachel Perman, Bronte Ray, Daniel Eric Smith, William Lee Starnes, Gregory Alan Stephens Jr., and Matthew Vines.

CA Epsilon – California Baptist University

Corresponding Secretary – James Buchholz; 264 Total Members; 14 New Members

New Initiates – Matthew Angeles, Blake Bayardo, Kyle Castillo, Nathan Castro, Garrett Deane, Matthew Diacono, Darren Gamble, Patrick McClung, Daisy Perez-Castaneda, Karmina Quichocho, Travis Smiley, Owen Sudds, Nicolas Vander Weide, and Mollyana Wood.

CT Beta – Eastern Connecticut State University

Corresponding Secretary – Dr. Mehdi Khorami; 515 Current Members; 14 New Members

New Initiates – Victoria Baker, James Calvert, Genevieve Fritsch, Fernanda Garcia Diaz, Brian Gonzalez, Amiah Jackson-Ward, Robert Johnson, Carolyn Kleinschmidt, Courtney Lusk, Erika Poremba, Brittany Romaniello, Stefanos Stravoravdis, Cory Struski, and Melissa Uribe.

CT Gamma – Central Connecticut State University

Chapter President – Nicholas Sabia; 70 Current Members

Other Spring 2020 Officers: Jonathan Maldonado, Vice President; Alyssa Mercaldi, Secretary; Sabrina Doolgar, Treasurer; Dr. Leah Frazee, Corresponding Secretary; and Dr. Marian Anton, Faculty Sponsor

Due to campus closures for COVID-19, we were unable to host our annual initiation ceremony. We intend to initiate new members eligible during the AY 2019-2020 in our initiation ceremony for AY 2020-2021.

GA Gamma – Piedmont College

Corresponding Secretary – Hope Menzel; 33 Total Members; 2 New Members

New Initiates – Haley Bolt and Geeth Mahagamage.

HI Alpha – Hawaii Pacific University

Corresponding Secretary – Tara Davis; 10 New Members

IA Alpha – University of Northern Iowa

Chapter President – Mariah Piippo; 1092 Total Members; 7 New Members

Other Spring 2020 Officers: Matthew Adams, Vice President; Ashley DeWis-

pelaere, Secretary; Stephanie Peiffer, Treasurer; and Dr. Mark D. Ecker, Corresponding Secretary and Faculty Sponsor

Our first spring KME meeting was held on February 27, 2020 in Wright Hall, where student member Matthew Adams presented his paper entitled “Magic Squares”. Unfortunately, our remaining scheduled KME meetings were cancelled due to the coronavirus outbreak in Iowa, as students were sent home after spring break. Three new members were initiated into KME this semester, however, no banquet was held.

New Initiates – Matthew Adams, Bethany Amos, Sydney DeBruin, Ashley DeWispelaere, Brynn Harberts, Kaelyn Koontz, Hannah Larsen, Derek Pape, Mariah Piippo, Matthew Roeding, and Jackson Twait.

IL Zeta – Dominican University

Corresponding Secretary – Mihaela Blanariu; 438 Total Members

We were excited to initiate nine new members and we were in the process of planning the ceremony, but we had to cancel the ceremony due to Covid-19. We are planning to initiate these nine new members along with other new members in the spring 2021.

IL Theta – Benedictine University

Chapter President – Emma Roberts

Other Spring 2020 Officers: Melissa Pickett, Vice President; Katie Farb, Secretary; Mariam Khan, Treasurer; Katelyn Beamish, SGA Rep/Alternate Officer; Dr. Thomas Wangler, Corresponding Secretary and Faculty Sponsor

IL Kappa – Aurora University

Corresponding Secretary – Dr. Sebastian Wyman; 59 Total Members; 31 New Members

New Initiates – James Petkus, Lindsey Arnold, Samantha Au, John Carl Balasbas, Elliot Barbel, Constance Birchall, Jonathan Birkey, Jennifer Carrillo, Nathaniel Chen, Shea Demonteverd, Thomas Doody, Casey Dycus, Alexis Eggen, Naomi Gomez, Bryan Guenther, Jessica Hertel, Kristina Krokosz, Nicole Lofshult, Jessika McCleskey, Areli Munoz, Maritza Navarrete, Leah Rachow, Elizabeth Rodriguez, Nicholas Rohl, Cole Rundle, Eduardo Soto, Christoph Stankus, Alexander Vargas, Darius Wheeler, Annika Wilson, and Bryan Zollinger.

KS Alpha – Pittsburg State University

Faculty Sponsor – Dr. Scott Van Thuong; 2148 Total Members

The Kansas Alpha section initiated 4 new members this past fall, bringing its total membership to 2148 members! Attendees at this meeting enjoyed pizza and math trivia. Unfortunately, COVID-19 resulted in a quiet spring semester, and the scheduled KME regional convention to be held at Pittsburg State University had to be canceled. We are looking forward to a

more active fall!

KS Beta – Emporia State University

Chapter President – Katherine Beckley; 1516 Total Members; 7 New Members

Other Spring 2020 Officers: Alec Bergeron, Vice President; Elisabeth Evans, Secretary; Amber Innes, Treasurer; Tom Mahoney, Corresponding Secretary; and Brian Hollenbeck, Faculty Sponsor

New Initiates – Rita Antwiwaa, Kelsey Bennett, Anastasia Chaplin, Kirstan DeVore, Rajarshi Dey, Paula Gálvez, and Hannah Showalter.

KS Delta – Washburn University

Chapter President – Jacob Talkin; 820 Total Members; 7 New Members

Other Fall 2017 Officers: ETCOther Spring 2020 Officers: Abby Bელი, Vice President; Madison Henley, Secretary; Mary Greene, Treasurer; Kevin Charwood, Corresponding Secretary and Faculty Sponsor

The Kansas Delta chapter of KME met twice with our math club during the spring semester and had presentations from guest speakers on both occasions. We initiated 7 new KME members on March 2, 2020 just before the COVID-19 pandemic forced the cancellation of any more in-person meetings for the remainder of the semester.

New Initiates – Manish Bhatta, Nikki Bolinger, Paul Flumen, Sophia Frick, Rebekah Homan, Caleb Niehues, and Lucas Paris.

KS Epsilon – Fort Hays State University

Corresponding Secretary – Jeffrey Sadler; 821 Total Members; 14 New Members

New Initiates –Bader Abukholdair, Edward Barfield, Seth Boxberger, Thomas Broxterman, Perla Camacho-Rosales, Thomas Dunn, Tanner Eiland, Nicole Fischer, Kevin Jones, Christian Meuli, Elle Stein, Judson Tillotson, Brianna Wooldridge, and Chelsea Zimmerman.

KS Eta – Sterling College

Corresponding Secretary – Amy Kosek

The Kansas Eta chapter of KME had planned to initiate new members and officers this spring and were unable to do so due to moving online for covid-19. We will do so in the 2020-2021 academic year.

MA Alpha – Assumption College

Corresponding Secretary – Joseph Alfano; 351 Current Members; 6 New Members

New Initiates – Michael Asante, Sarah Haak, Veronica Johnson, Angela Martinez, Cassandra Saniuk, and Corey Soper.

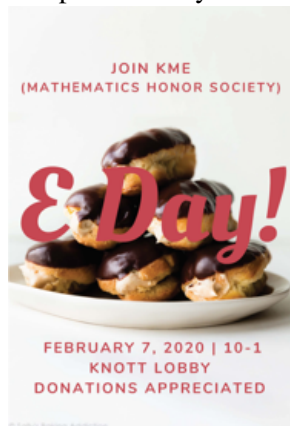
MD Alpha – Notre Dame of Maryland University

Chapter President – Amanda Ashton; 397 Current Members; 4 New Members

Other Spring 2020 Officers: Hannah Campbell, Vice President; Emily

Garzon, Secretary; Aisha Azhar, Treasurer; Charles Buehrle, Corresponding Secretary and Faculty Sponsor

Below is a flyer from MD Alpha's "e Day" event.



New Initiates – Vanessa Dunn, Ericka Kaschak, Breonna Morten, and Rachel Steinberg.

MD Delta – Frostburg State University

*Chapter President – Jordan Thomas; 537 Total Members; 8 New Members
Other Spring 2020 Officers: Katelynn Suesse, Vice President; Bailey Brew, Secretary; Chad Shumaker, Treasurer; Mark Hughes, Corresponding Secretary and Faculty Sponsor; and Frank Barnet, Faculty Sponsor*

Maryland Delta Chapter held a meeting in late February where we played games, had pizza, and planned for the semester's activities. At our Initiation Ceremony on March 1 we welcomed eight new members to the chapter. After the ceremony, faculty sponsor Dr. Frank Barnet gave a presentation on using Mathematica to construct 3D anaglyphs. Our bake sale during the second week of March was quite successful due to the great help from several chapter members. A few days later, the last day of classes before spring break was cancelled due to the coronavirus pandemic. Following spring break classes moved to online instruction and Maryland Delta Chapter suspended all activities. We hope to resume some activities in the fall. We offer our best wishes to Treasurer Chad Shumaker who graduated in May.

New Initiates – Nathan Abankwah, Ashley Armbruster, Jay Collins, Madison Green, Eliza Griffith, Robert Hood, Andrew Kastner, and Julia Moore.

MI Beta – Central Michigan University

Chapter President – Natalie DeVos; 1754 Current Members; 4 New Members

Other Spring 2020 Officers: Austin Konke, Vice President; Emily Naegelin, Secretary; Evan Miller, Treasurer; Camilla Madacki, Public Relations;

and Dr. Ben Salisbury, Corresponding Secretary and Faculty Sponsor

During the 2019–2020 academic year, KME met once every two weeks on average. The following is a list of the events which took place during the Fall 2019 and Spring 2020 semesters. For Fall 2019, KME participated in the CMU MainStage event at the start of the semester. The first meeting of the semester was held on September 3 and featured several ice breakers and opportunities for new and old members to become acquainted. KME held a book sale in from September 16 until September 18 to raise funds for their activities. On September 14, KME Secretary Emily Naegelin used simple crafts to demonstrate the mathematical patterns in nature. On October 1, KME Vice President Austin Konkel and Treasurer Evan Miller presented on their results from summer research and discussed their experience with research in mathematics. On October 15, KME invited guest speaker Professor Emeritus Robert Chaffer to give a talk about his artwork and the mathematical ideas he uses to create his pieces. On October 29, the meeting was devoted to the interests of the E-Board. Each officer gave a brief presentation on a mathematical idea that intrigues them. With the semester winding down, KME held a game night on November 12. The game was mathematical Jeopardy! Math-a-palooza was co-organized by KME and the AMS Graduate Student Chapter. The Fall 2019 event was held on December 6. For Spring 2020, the first meeting was held on January 21 where members played Math Bingo for a chance to win some prizes. KME's biannual book sale fundraiser was held from January 28 to January 30 to raise money for the organization. On February 4, KME members worked together to solve brain teasers and logical puzzles by playing Mind Trap. On February 18, guest speaker Professor Tibor Marcinek gave a workshop on GeoGebra involving some fascinating properties of conic sections. On March 3, KME enjoyed a fun mathematical scavenger hunt, working together to solve clues in and around the math building. Due to the coronavirus and the subsequent move to online learning, the remaining meetings and events (including a research talk from a KME member, a game night, the Initiation Ceremony, and the Math-a-palooza tutoring) were cancelled.

New Initiates – Camilla Madacki, Robert Mason, Christina Millikin, and Emily Naegelin.

MI Delta – Hillsdale College

Corresponding Secretary – Dr. David Gaebler; 350 Total Members; 10 New Members

New Initiates – Corresponding Secretary – Dr. David Gaebler; 350 Total Members; 10 New Members

MI Epsilon – Kettering College

Chapter President – Danny Boyle (A) and Pietro Pellerito (B); 928 Total

Members; 32 (Summer Section A) and 50 (Fall Section B) New Members and 33 (Winter Section A)

Other Fall 2019 and Spring 2020 Officers: Makayla Carpenter (A) and Alley Broom (B), Vice President; Lindsey Malston (A) and Mary Allen (B), Secretary; Rebecca Abbott-Mccune (A) and Charles Cook (B), Treasurer; and Dr. Boyan Dimitrov, Corresponding Secretary and Matthew Causle, Faculty Sponsor

Kettering has now successfully re-established both its A and B-section chapters of KME. We were very pleased to add 50 members to B-section last fall, and downright elated to initiate new 32 members this spring! Future events that are planned include social mixers, tutoring high school students in the greater Flint community, and perhaps a T-shirt design to raise funds for the society. Actually, the winter ceremony for A-section is currently being planned. We have a T-shirt design competition, and perhaps a bake sale this semester.

The Summer and Fall terms brought lots of excitement into our Mathematics life. First of all, our Actuarial program was granted the Advanced Curriculum designation by the Society of Actuaries thanks to the work of our colleague Hee Seok Nam. He joined Kettering 4 years ago, and his job is getting highest ratings.

Next I would notice the continued use of the “Islands” virtual environment in our Statistics education. This program is supported by a Terrell grant through our Center of Excellence in Teaching and Learning (CETL) and initiated by our Department Head, Dr. Leszek Gawarecki). The “Islands” offers a holistic experience for students (and for teaching instructors also!): they transition through all stages of a real statistical study, from planning, data collection and organization, to formulating and testing hypotheses and drawing conclusions and making recommendations. Final reports are in the form of class presentations. Students are excited about the new “toy”. We (the faculty) presented our results in the Fall Lilly conference in Travers City, the QuadPod Symposium. The presentation was met with great interest on behalf of the Conference participants. An article is already published in the Proceedings of the LILI Conferences. Maybe at some of the future KME traditional sessions our students will report their results and experience in learning statistics.

Another deserving news is that our Kettering Mathematics Olympiad is getting popularity and enthusiastic participation. The Kettering Mathematics Olympiad is a competition designed to identify and encourage students with interests and abilities in mathematics. Our goal is to host this Olympiad that is one of the most prestigious mathematical competitions in the region. The examination is designed for students in grades 9 through

12. Students working towards a high school degree who is currently enrolled in a public school, private school or a home-school program could sit for the examination. The competition consists of six challenging problems and has a time limit of four hours. The problems range from “mind-benders” that require little mathematical skills to problems that require the knowledge of geometry, trigonometry and beginning calculus. Here are the 2019 Winners:

- First – Kenta Suzuki
- Second – Andrew Zhou
- Third – Ross Gao
- Fourth – Seventh Place (In alphabetical order): Giovanni Cavataio, Noah Ferguson, Shubhan Nagarkar, Noah Ray
- Eighth – Fifteenth Place (In alphabetical order): Kalil Black, Matthew Ejakov, Gabriel Howald, Benjamin Maynard, David Kies, Ryan Mozariwskyj, Jack Taylor, Andy Yao

Congratulations, and Cheers! And here are the Prizes for the Winners. Prize winners are chosen from the combined participant pool from the two locations. Top Prizes:

- First Place – a 4 1/2 -year, full-tuition scholarship to Kettering University
- Second Place – a 4 1/2 -year, full-tuition scholarship to Kettering University
- Third Place – a 4 1/2 -year, full-tuition scholarship to Kettering University
- Fourth to Seventh Place – a 4 1/2 -year, half-tuition scholarship to Kettering University
- Eighth-Fifteenth place – a 4 1/2 -year, \$3000 scholarship to Kettering University. (Starting 2019)

The scholarship portions of the prizes are awarded upon admission to Kettering University. Recipients of these scholarships are not eligible for any other scholarships awarded by Kettering University.

Also it makes sense to point out, that Kettering has a lots of other initiatives, like Robotic Competitions (Local and Nationwide, The outdoor Lab for auto-drive vehicles, etc.). I cannot list them all.

Something new and remarkable: The president Dr. Robert McMahan

started an additional building that will enlarge the comfort of studying, teaching and relaxation of students, staff and visitors, but about this – in my next report.

I finish my Fall 2019 report with some photos of our B-section Kettering KME Michigan Epsilon new KME student members in the Fall 2019. Thanks for the pictures Prof. Hee Seok Nam!



All the initiates on the ceremony podium. At the table are Faculty Sponsor, Professor Dr. Matt Causley and the Corresponding secretary, Professor Dr. B. Dimitrov.



Corresponding secretary congratulates Demetry Blackwood for his KME initiation.

We had a winter term initiation for students in our Section A, which brought in over 30 new members. We also had a T-shirt design competition that semester, but final results were not made due to the changes in Institutional policy, such as I suppose were introduced elsewhere in the US. Below is the group picture of our Section A new initiated students and some faculty KME involved. Picture was taken by Hee Seok Nam, our newest Math Faculty colleague, and one of the most enthusiastic workers in the field of

Actuarial teaching and research.



The Spring Term started completely under the new conditions of online teaching. Our faculty had the last 2 weeks of the Winter Term experiencing this new technology of teaching and 2 intermission weeks to prepare it better. It seems that our Spring Term will go completely online. However, we have some achievements, and I took it from our half year Report to the University Board of Trustees.

Community Engagement: Dr. Hee Seok Nam engaged with Curiosity Academy in Flint to provide activities involving a virtual environment “The Islands.”

Dr. Boyan Dimitrov engaged a student group to participate in the Flint Student Research Conference, held by the Office of Research and Sponsored Programs at University of Michigan-Flint. 5 students (Edward Wenzel, Kyle Redies, Zach Barhite, Jacob Burger, and Joe Saval) presented their Course project from “The Islands,” called “How does Gender and Weight affect Balance while intoxicated?”

Professional Development: Dr. L. Gawarecki, Dr. Hee Seok Nam, Dr. B. Dimitrov and Prof. G. Rablau presented at CETL’s Tuesday Teaching Talk “Enhancing Student Engagement in Statistics using “The Islands” Virtual Environment.”

Student Support: Mathematics Department introduced ALEKS (Assessment and Learning Knowledge Spaces) for math placement for freshmen and as a study tool for improving freshmen scores. Mathematics faculty hold “Help Room” available for all students. This complements regular student hours.

Math Department faculty have had a total of 4 published articles since January 2020. And according to the Research Gate, the corresponding secretary Dr B. Dimitrov reached a milestone. Here is the text; Great job, Boyan! Your research items reached 500 citations, achieved on April 22, 2020.

New Initiates – Rebecca Sag Abbott-Mccune, John Allen, Nicholas Wray Allston, Daniel Patrick Boyle, Olivia Rose Bussone, Makayla R. Carpenter, Chi Kin Choy, Tyler Allen

Duff, Juan Javier Espino, Jeremy Michael Gooch, Sarah Yilin He, Connor Garrett Keais, Mikaela K. Kelley, Abigail L. Lund, Mark David Lutz, Lindsey K. Malson, Nicholas A. Mitchell, Matthew Ryan Moore, Joshua James Nelson, Emilia Nwakerendu, Taylor John Plekker, Emily Joelle Poirier, Adriana C. Raffin, Cole A. Schaar, Savannah Xiaojing Sojourn, Jessica Erin Stanley, Tara Nicole Tepin, Chase Michael Vondrak, Kasimir Joseph Waitkus, Mckensie Rico Winn, Ryan Ciric, and Andrew Williams.

MN Alpha – Metropolitan State University

Corresponding Secretary – Rikki Wagstrom; 17 Current Members; 17 New Members

New Initiates – Matthew Ambers, Carter Clark, Jason Evans, Claire Falvey-Sander, Alia Grafenstein, Sarah Hartzel, Thaddeus Hoffman, Joshua Illg, Katherine Johnson, Cindy Kaus, Eliahou Maruani, Ross Radtke, Matthew Rose, Brian Strand, Alec Timmerman, Rikki Wagstrom, and Andrew Wahl.

MO Beta – University of Central Missouri

Corresponding Secretary – Dr. Rhonda McKee; 1529 Total Members; 9 New Members

New Initiates – Heather Buckingham, Haleigh Clark, Halie Tyne Gelatko, Christian C. Hirni, William Kussmaul, Claire E. Myers, Lennon Puls, Jacob Lakinger, Dessie Shimel, and Dustin Luke Stricklin.

MO Epsilon – Central Methodist University

Corresponding Secretary – Pam Gordy

Due to the COVID-19 pandemic, initiation was postponed until the fall semester.

MO Theta – Evangel University

*Chapter President – Peter Russell; 290 Total Members; 3 New Members
Other Spring 2020 Officers: Hannah Tower, Vice President; and Don Tosh,
Corresponding Secretary and Faculty Sponsor*

Meetings were held in January and February. In January we initiated 3 new members and elected new officers. Further meetings for the semester and our planned trip to the regional convention in Pittsburg, KS, were cancelled when all of our classes went online due to the pandemic.

New Initiates – Daniel Braunberger, Peter Russell, and Hannah Alexandra Tower.

MO Kappa – Drury University

Corresponding Secretary – Dr. Carol Browning; 311 Total Members; 9 New Members

New Initiates – Danni Angell, Austin Froese, Gwynna Fuller, Katherine Glaze, Brooke Kempf, Zachary Mason, Megan Rice, Walter Rowe, and Sydney Thompson.

MO Mu – Harris-Stowe State University

Corresponding Secretary – Ann Podleski; 108 Total Members; 7 New Members

We had a virtual initiation ceremony (using Zoom) and had approximately

40 attendees. In addition to this being our first virtual ceremony, the seven initiates into the Missouri Mu Chapter of KME included the first Biology major to be initiated, and the first initiate in the joint Math/Engineering major in our partnership program with St. Louis University Parks College of Engineering. Both these students are May 2020 graduates. Two other initiates are also May 2020 graduates, one in Secondary Mathematics Education and a Mathematics major with strong interest in Computer Science. Although our KME chapter has not been active in the past 5 years, our math program has evolved to include interdisciplinary opportunities for math majors at Harris-Stowe State University and we have recently added an interdisciplinary degree program in our department - Sustainability and Urban Ecology with different concentrations including one with math/physics/computational focus.

MO Xi – William Woods University

Corresponding Secretary – Dr. Chris Schneider; 19 Total Members; 4 New Members

New Initiates – Elizabeth Kasubke, Zachary Knopf, Karyna Sagalai, and Brittany Wonderly.

MS Alpha – Mississippi University for Women

Corresponding Secretary – Dr. Joshua Hanes

Due to covid-19 we were unable to meet this semester. We hope to have initiations and office elections in the fall.

MS Epsilon – Delta State University

Corresponding Secretary – Lee Virden; 121 Total Members; 4 New Members

New Initiates – Angelise Campbell, Chasten Conway, Alison Fullilove, and Paxton Stratton.

NC Epsilon – North Carolina Wesleyan College

Corresponding Secretary – Bill Yankosky; 101 Total Members; 9 New Members

New Initiates – Alyssa Nichole Brookhart, Dion Cox Jr. , Cody Fogleman, Maria Joao Gaspar, Juri Kim, Jovan Damani Pope, Jadéjah Robinson, Michelle Sackey-Ansha, and Rajan Shah.

NE Alpha – Wayne State College

Chapter Co-Presidents – Morgan Barner and Brittani Ludwig; 1039 Total Members; 4 New Members

Other Spring 2020 Officer: Dr. Jennifer Langdon, Corresponding Secretary and Faculty Sponsor

Nebraska Alpha KME members hosted two board game nights for math majors and other members of campus, total attendance about 40. Held the annual Pi Day fundraiser - KME along with Math Club baked and deco-

rated cookies in the shape of Pi and cupcakes that looked like berry pies. These were sold around campus to students, faculty, and staff. (Luckily students decided to do this early, before our campus went to remote learning due to COVID-19.) The chapter had plans to attend two conferences, and host two speakers. These plans were thwarted by the Coronavirus.

NE Beta – University of Nebraska Kearney

Chapter President – Tiffany Collins; 922 Total Members

Other Spring 2020 Officers: Joshua Garcia, Vice President; Evan Olson, Secretary; Julie Kent, Treasurer; and Dr. Katherine Kime, Corresponding Secretary and Faculty Sponsor

Plans for Spring 2020 were upended as our last day of face-to-face classes was March 17. Thus there was no initiation, and the cookie dough sale could not proceed. Corbin Snow had planned to give a presentation at the Regional Convention, which was cancelled, and was planning to submit a PowerPoint version. Seniors Tiffany Collins, Joshua Garcia, Evan Olson, Carli Pofahl, Christian Schleif, and Corbin Snow were to graduate. Evan was accepted into the mathematics graduate program at Kansas State University.

NE Delta – Nebraska Wesleyan University

Chapter President – Drew Damme; 293 Total Members; 8 New Members

Other Spring 2020 Officers: Samantha Wright, Vice President; Melissa Erdmann, Faculty Sponsor and Corresponding Secretary

Due to the Corona virus, it was a pretty slow spring. We didn't hold our annual Pi Mile fun run nor initiation/picnic as usual. Although there were no new members initiated during the Spring 2020 semester, we have eight who will be initiated in the fall semester, if all goes well.

New Initiates – Andrew Beutler, Baylie Oliver, Thomas Robertson, Logan Rowe, Hudson Son, Samuel Streeter, Ann Truka, and Vanessa Wergin.

NH Alpha – Keene State College

Corresponding Secretary – Vincent J. Ferlini; 312 Total Members; 17 New Members

New Initiates – Lydia Ahlstrom, Kathryn Anderson, Nicole Berry, Abigail Call, Ashley Chopelas, Alison Daisy, Courtney DesRoches, Ally McCall, Erin McNichols, Meghan Moran, Lottie Page, Andrew Parsons, Madison Pawela, Faith Pudlo, Daniel Rearick, Jackson Turni, and Danielle Wiley.

NY Iota – Wagner College

Corresponding Secretary – Marisa Scarpa; 400 Total Members; 7 New Members

New Initiates – Autumn Brianna Clark, Nicolette Cunsolo, Noelle Harvey, Kaela Schrier,

Rosa Schuchert, Mohamed Essam Lotfi Topala, and Jenna D. Zerino.

NY Mu – St. Thomas Aquinas College

Corresponding Secretary – Dr. Heather A. Rave; 217 Total Members; 6 New Members

New Initiates – Tara Fears, Manuel Gutierrez, Melvin John, Danielle Nicole Lieberman, Jessica Alexandra Ligeiro, and Evert D. Magana.

NY Xi – Buffalo State College

Corresponding Secretary – Jane Cushman; 47 Total Members; 8 Members

New Initiates – Vincent P. Argo, Michael D. Capestrani, Tyler E. Federson, Dawn A. Jones, Brendan J. Kendall, Lauren Krug, Elizabeth Jane Nash, and Abigail C. Webb.

NY Pi – Mount Saint Mary College

Corresponding Secretary – Dr. Lee Fothergill; 129 Total Members; 6 New Members

New Initiates – Sarah Canzone, Nicole M. Cervone, Myah Alana Dewitt, Elizabeth A. Halpin, Taylor V. Moreau, and Mikayla A. Pileggi.

NY Rho – Molloy College

Corresponding Secretary – Manyiu Tse and Deborah Upton; 188 Total Members; 1 New Member

New Initiate – Jillian Mayr.

OH Gamma – Baldwin Wallace University

Corresponding Secretary – David Calvis; 50 Current Members

Due to the pandemic, our spring 2020 initiation was postponed until fall. Students were told that anyone who was eligible to join would remain so regardless of academic outcome this term.

OH Eta – Ohio Northern University

Corresponding Secretary – Ryan Rahrig; 476 Total Members; 1 New Member

New Initiates – Patrick Baker, Joseph Bement, Lillian Brautigam, Andrew Cooley, Caroline Goeller, Nathan Hagerdorn, Steven Manns, and William Sierzputowski.

OH Theta – Capital University

Chapter President – Taylor Manivanh; 54 Total Members

Other Spring 2020 Officers: Colin Swisher, Vice President; Joshua Wissman, Secretary; Jarrett Williams, Treasurer; Paula Federico, Corresponding Secretary; and Jon Stadler, Faculty Sponsor

During the Fall 2019 semester our Chapter of KME semester held two meetings where they started planning events for the Spring (Pi-day and KME t-shirts). During the Spring semester students met several times and planned our Pi-day event. This event included handing out mini pies, a talk by guest speaker Prof. Joy Longfellow, and wearing Pi-day t-shirts designed by our students. Unfortunately, classes at Capital University were cancelled two days before our event planned for Friday March 13. The

pies were donated to a local Food Pantry. We were also planning our initiation ceremony for the first week of April and we decided to postpone to early Fall. In the case that we are not able to hold a face-to-face ceremony in August or September, we will organize a virtual ceremony. Some of our graduating members were able to stop by Campus to pick up cords before the virtual graduation ceremony (pictures below). The remaining cords will be mailed to students. The Chapter selected Josh Wissman as president for AY 20-21. The remaining positions will be filled after welcoming new members in the Fall.



PA Theta – Susquehanna University

Corresponding Secretary – Kenneth Brakke; 587 Total Members; 13 New Members

New Initiates – Erica Getz, Mara Hashuga, Megan Herbine, Jenna Juntunen, Jocelyn McMahon, Kyle Miller, Stephen Oravec, Suyash Pandey, John Pellock, Micaela Pozo Castro, Lindsey Pugh, Joseph Tomasetti, and Cameron Wolf.

PA Kappa - Holy Family University

Chapter President – Melissa Cahill; 167 Total Members

Other Spring 2020 Officer: Sister Marcella Louise Wallowicz CSFN, PhD, Corresponding Secretary and Faculty Sponsor

Due to Covid-19, the university went online, effective March 16, 2020. All activities and events were cancelled, including the initiation. It will be rescheduled for Fall, 2020.

PA Lambda – Bloomsburg University

Corresponding Secretary – Eric B. Kahn; 762 Total Members; 3 New

Members

New Initiates – Lindsey Boutin, Jacob Snyder, and Cameron Stouffer.

PA Mu – Saint Francis University

Chapter President – Teresa Reid; 483 Total Members

Other Spring 2020 Officers: Kari Lagan, Vice President; Katherine Augustine, Secretary; Ryan Alu, Treasurer; Dr. Brendon LaBuz, Corresponding Secretary and Faculty Sponsor

The Pennsylvania Mu Chapter of Kappa Mu Epsilon invited ten students to join the honor society in Spring 2020. Unfortunately, our campus moved online before we could hold an initiation ceremony due to the pandemic. We intend to hold our ceremony once we are back on campus, hopefully this fall.

PA Xi – Cedar Crest College

Corresponding Secretary – Dr. Joshua Harrington; 119 Total Members; 3 New Members

New Initiates – Breille Duncan, Emily Klenk, and Paige Dinbokowitz.

PA Rho – Thiel College

Chapter President – Breanna Mesich; 1136 Total Members; 4 New Members

Other Spring 2020 Officers: Taylor Guth, Vice President; Courtney Hariman, Secretary; Emily Groves, Treasurer; Dr. Jie Wu, Faculty Sponsor; and Dr. Russell Richins, Corresponding Secretary

We had several activities planned for spring semester, but all of them were canceled due to COVID-19. The planned activities were a chocolate pi sale on pi day (actually day before since pi day was a Saturday this year), Challenge 24 and food drive scheduled for the next week, and an in person initiation of new members sometime in April.

New Initiates – Kara Baumgardner, Cassandra Bown, Justin Olah, and Ethan Stishan.

PA Tau – DeSales University

Corresponding Secretary – Annmarie Houck; 86 Total Members; 7 New Members

New Initiates – Alessio Brattoli, Justin S. Cline, Sarah E. Del Grande, Anthony J. Hessler, Samantha M. Mullin, Jaime M. Oldford, and Taylor J. Winner.

RI Beta – Bryant University

Chapter President – Christopher Ethier; 171 Total Members; 13 New Members

Other Spring 2020 Officers: Constance Tang, Vice President; Alexandra Sherman, Secretary; Liam Mahler, Treasurer; Prof. John Quinn, Corresponding Secretary; and Prof. Alan Olinsky, Faculty Sponsor

We had a student prepare a video presentation for the virtual regional conference hosted by Molloy College during the spring 2020. Also, we had

planned to host the New England KME Conference in spring 2022, but we realize that the situation might have changed due to the fact this spring's regional conference was not able to be held on-campus at Molloy. We were unable to have an on-campus KME initiation this spring, but we were able to welcome 13 new students this spring.

SC Epsilon – Francis Marion University

Corresponding Secretary – Dr. Daniel Scofield; 139 Total Members; 9 New Members

New Initiates – Astou Aw, Taylor Boatwright, Harley Collins, Monica Cox, Thomas Owen Dixon, Kody Lawler, Philip Mickel, James Morris, and Thomas Schnibben.

TN Gamma – Union University

Chapter President – Jenna Dula; 505 Total Members

Other Spring 2020 Officers: Josie Carrier, Vice President; Ainsley Duncan, Secretary and Treasurer; John Mayer, Webmaster and Historian; Bryan Dawson, Corresponding Secretary; and Matt Lunsford, Faculty Sponsor

TX Eta – Hardin-Simmons University

Corresponding Secretary – Jessica Rieger; 329 Total Members; 7 New Members

New Initiates – Bohner Cottongame, Keely Eshelman, Joseph Gilligan, Matthew McCartney, Wei Chin Neoh, Caleb Rosenblad, and Andrea Strohl.

TX Iota – McMurry University

Corresponding Secretary – Dr. Kelly L. McCoun; 301 Total Members; 3 New Members

New Initiates – Yuwei Bao, Angelica Navarro, and Joseph Watson.

TX Lambda – Trinity University

Corresponding Secretary – Dr. Hoa Nguyen; 290 Current Members

Due to the pandemic, no new initiations were held this semester.

WI Alpha – Mount Mary University

Corresponding Secretary – Sherrie Serros; 4 Current Members; 4 New Members

Other Spring 2020 Officer: Elysium Pajhuab, Vice President

On April 30, 2020, Wisconsin Alpha chapter held a virtual Zoom initiation ceremony for students Hannah Ashbach, Melissa Golo and Aubrey Stoehr, and faculty member Jeremy Edison. The ceremony was attended by faculty, students and family in five states. After the formal initiation, new initiate, Dr. Jeremy Edison, gave a presentation, "Nothing with a Twist: An exploration of infinity."

New Initiates – Hannah Ashbach, Jeremy Edison, Melissa Golo, and Aubrey Stoehr.

WV Alpha – Bethany College

Chapter President – Rachel E. Gantzer; 184 Total Members; 6 New Mem-

bers

Other Spring 2020 Officers: Joseph A. Makowski, Vice President; Ethan J. Young, Secretary/Treasurer; and Dr. Adam C. Fletcher, Corresponding Secretary and Faculty Sponsor

It has been a strange year for the West Virginia Alpha chapter, as it has for everyone. Just before Spring Break, and COVID-19 directed the College into distance-delivery mode, the chapter assisted the campus Mathematics and Computer Science Club in hosting the fourteenth annual Math/Science Day competition for local high school students on campus. The chapter also supported Bethany College's virtual initiation ceremony of three new members into the Upsilon Pi Epsilon international computing sciences honor society, while also holding its first virtual initiation ceremony for two new members. The chapter was looking forward to attending the Great Lakes Regional convention this spring, but are eagerly anticipating next year's National Biennial convention.

New Initiates -- Michael Aaron Ceraolo, David Tome del Arco, Lena A. Grogan, Joseph Alexander Makowski, Becca Lyn Theaker, Ethan James Young.

Active Chapters of Kappa Mu Epsilon

Listed by date of installation

Chapter	Location	Installation Date
OK Alpha	Northeastern State University, Tahlequah	18 Apr 1931
IA Alpha	University of Northern Iowa, Cedar Falls	27 May 1931
KS Alpha	Pittsburg State University, Pittsburg	30 Jan 1932
MO Alpha	Missouri State University, Springfield	20 May 1932
MS Alpha	Mississippi University for Women, Columbus	30 May 1932
NE Alpha	Wayne State College, Wayne	17 Jan 1933
KS Beta	Emporia State University, Emporia	12 May 1934
AL Alpha	Athens State University, Athens	5 Mar 1935
NM Alpha	University of New Mexico, Albuquerque	28 Mar 1935
IL Beta	Eastern Illinois University, Charleston	11 Apr 1935
AL Beta	University of North Alabama, Florence	20 May 1935
AL Gamma	University of Montevallo, Montevallo	24 Apr 1937
OH Alpha	Bowling Green State University, Bowling Green	24 Apr 1937
MI Alpha	Albion College, Albion	29 May 1937
MO Beta	University of Central Missouri, Warrensburg	10 Jun 1938
TX Alpha	Texas Tech University, Lubbock	10 May 1940
KS Gamma	Benedictine College, Atchison	26 May 1940
IA Beta	Drake University, Des Moines	27 May 1940
TN Alpha	Tennessee Technological University, Cookeville	5 Jun 1941
MI Beta	Central Michigan University, Mount Pleasant	25 Apr 1942
NJ Beta	Montclair State University, Upper Montclair	21 Apr 1944
IL Delta	University of St. Francis, Joliet	21 May 1945
KS Delta	Washburn University, Topeka	29 Mar 1947
MO Gamma	William Jewell College, Liberty	7 May 1947
TX Gamma	Texas Woman's University, Denton	7 May 1947
WI Alpha	Mount Mary College, Milwaukee	11 May 1947
OH Gamma	Baldwin-Wallace College, Berea	6 Jun 1947
MO Epsilon	Central Methodist College, Fayette	18 May 1949
MS Gamma	University of Southern Mississippi, Hattiesburg	21 May 1949
IN Alpha	Manchester College, North Manchester	16 May 1950
PA Alpha	Westminster College, New Wilmington	17 May 1950
IN Beta	Butler University, Indianapolis	16 May 1952
KS Epsilon	Fort Hays State University, Hays	6 Dec 1952
PA Beta	LaSalle University, Philadelphia	19 May 1953
VA Alpha	Virginia State University, Petersburg	29 Jan 1955
IN Gamma	Anderson University, Anderson	5 Apr 1957
CA Gamma	California Polytechnic State University, San Luis Obispo	23 May 1958
TN Beta	East Tennessee State University, Johnson City	22 May 1959
PA Gamma	Waynesburg College, Waynesburg	23 May 1959
VA Beta	Radford University, Radford	12 Nov 1959
NE Beta	University of Nebraska—Kearney, Kearney	11 Dec 1959
IN Delta	University of Evansville, Evansville	27 May 1960
OH Epsilon	Marietta College, Marietta	29 Oct 1960
MO Zeta	University of Missouri—Rolla, Rolla	19 May 1961

NE Gamma	Chadron State College, Chadron	19 May 1962
MD Alpha	College of Notre Dame of Maryland, Baltimore	22 May 1963
CA Delta	California State Polytechnic University, Pomona	5 Nov 1964
PA Delta	Marywood University, Scranton	8 Nov 1964
PA Epsilon	Kutztown University of Pennsylvania, Kutztown	3 Apr 1965
AL Epsilon	Huntingdon College, Montgomery	15 Apr 1965
PA Zeta	Indiana University of Pennsylvania, Indiana	6 May 1965
TN Gamma	Union University, Jackson	24 May 1965
IA Gamma	Morningside College, Sioux City	25 May 1965
MD Beta	McDaniel College, Westminster	30 May 1965
IL Zeta	Dominican University, River Forest	26 Feb 1967
SC Beta	South Carolina State College, Orangeburg	6 May 1967
PA Eta	Grove City College, Grove City	13 May 1967
NY Eta	Niagara University, Niagara University	18 May 1968
MA Alpha	Assumption College, Worcester	19 Nov 1968
MO Eta	Truman State University, Kirksville	7 Dec 1968
IL Eta	Western Illinois University, Macomb	9 May 1969
OH Zeta	Muskingum College, New Concord	17 May 1969
PA Theta	Susquehanna University, Selinsgrove	26 May 1969
PA Iota	Shippensburg University of Pennsylvania, Shippensburg	1 Nov 1969
MS Delta	William Carey College, Hattiesburg	17 Dec 1970
MO Theta	Evangel University, Springfield	12 Jan 1971
PA Kappa	Holy Family College, Philadelphia	23 Jan 1971
CO Beta	Colorado School of Mines, Golden	4 Mar 1971
KY Alpha	Eastern Kentucky University, Richmond	27 Mar 1971
TN Delta	Carson-Newman College, Jefferson City	15 May 1971
NY Iota	Wagner College, Staten Island	19 May 1971
SC Gamma	Winthrop University, Rock Hill	3 Nov 1972
IA Delta	Wartburg College, Waverly	6 Apr 1973
PA Lambda	Bloomsburg University of Pennsylvania, Bloomsburg	17 Oct 1973
OK Gamma	Southwestern Oklahoma State University, Weatherford	1 May 1973
NY Kappa	Pace University, New York	24 Apr 1974
TX Eta	Hardin-Simmons University, Abilene	3 May 1975
MO Iota	Missouri Southern State University, Joplin	8 May 1975
GA Alpha	State University of West Georgia, Carrollton	21 May 1975
WV Alpha	Bethany College, Bethany	21 May 1975
FL Beta	Florida Southern College, Lakeland	31 Oct 1976
WI Gamma	University of Wisconsin—Eau Claire, Eau Claire	4 Feb 1978
MD Delta	Frostburg State University, Frostburg	17 Sep 1978
IL Theta	Benedictine University, Lisle	18 May 1979
PA Mu	St. Francis University, Loretto	14 Sep 1979
AL Zeta	Birmingham-Southern College, Birmingham	18 Feb 1981
CT Beta	Eastern Connecticut State University, Willimantic	2 May 1981
NY Lambda	C.W. Post Campus of Long Island University, Brookville	2 May 1983
MO Kappa	Drury University, Springfield	30 Nov 1984
CO Gamma	Fort Lewis College, Durango	29 Mar 1985
NE Delta	Nebraska Wesleyan University, Lincoln	18 Apr 1986
TX Iota	McMurry University, Abilene	25 Apr 1987
PA Nu	Ursinus College, Collegeville	28 Apr 1987
VA Gamma	Liberty University, Lynchburg	30 Apr 1987

NY Mu	St. Thomas Aquinas College, Sparkill	14 May 1987
OH Eta	Ohio Northern University, Ada	15 Dec 1987
OK Delta	Oral Roberts University, Tulsa	10 Apr 1990
CO Delta	Mesa State College, Grand Junction	27 Apr 1990
PA Xi	Cedar Crest College, Allentown	30 Oct 1990
MO Lambda	Missouri Western State College, St. Joseph	10 Feb 1991
TX Kappa	University of Mary Hardin-Baylor, Belton	21 Feb 1991
SC Delta	Erskine College, Due West	28 Apr 1991
NY Nu	Hartwick College, Oneonta	14 May 1992
NH Alpha	Keene State College, Keene	16 Feb 1993
LA Gamma	Northwestern State University, Natchitoches	24 Mar 1993
KY Beta	Cumberland College, Williamsburg	3 May 1993
MS Epsilon	Delta State University, Cleveland	19 Nov 1994
PA Omicron	University of Pittsburgh at Johnstown, Johnstown	10 Apr 1997
MI Delta	Hillsdale College, Hillsdale	30 Apr 1997
MI Epsilon	Kettering University, Flint	28 Mar 1998
MO Mu	Harris-Stowe College, St. Louis	25 Apr 1998
GA Beta	Georgia College and State University, Milledgeville	25 Apr 1998
AL Eta	University of West Alabama, Livingston	4 May 1998
PA Pi	Slippery Rock University, Slippery Rock	19 Apr 1999
TX Lambda	Trinity University, San Antonio	22 Nov 1999
GA Gamma	Piedmont College, Demorest	7 Apr 2000
LA Delta	University of Louisiana, Monroe	11 Feb 2001
GA Delta	Berry College, Mount Berry	21 Apr 2001
TX Mu	Schreiner University, Kerrville	28 Apr 2001
CA Epsilon	California Baptist University, Riverside	21 Apr 2003
PA Rho	Thiel College, Greenville	13 Feb 2004
VA Delta	Marymount University, Arlington	26 Mar 2004
NY Omicron	St. Joseph's College, Patchogue	1 May 2004
IL Iota	Lewis University, Romeoville	26 Feb 2005
WV Beta	Wheeling Jesuit University, Wheeling	11 Mar 2005
SC Epsilon	Francis Marion University, Florence	18 Mar 2005
PA Sigma	Lycoming College, Williamsport	1 Apr 2005
MO Nu	Columbia College, Columbia	29 Apr 2005
MD Epsilon	Stevenson University, Stevenson	3 Dec 2005
NJ Delta	Centenary College, Hackettstown	1 Dec 2006
NY Pi	Mount Saint Mary College, Newburgh	20 Mar 2007
OK Epsilon	Oklahoma Christian University, Oklahoma City	20 Apr 2007
HA Alpha	Hawaii Pacific University, Waipahu	22 Oct 2007
NC Epsilon	North Carolina Wesleyan College, Rocky Mount	24 Mar 2008
NY Rho	Molloy College, Rockville Center	21 Apr 2009
NC Zeta	Catawba College, Salisbury	17 Sep 2009
RI Alpha	Roger Williams University, Bristol	13 Nov 2009
NJ Epsilon	New Jersey City University, Jersey City	22 Feb 2010
NC Eta	Johnson C. Smith University, Charlotte	18 Mar 2010
AL Theta	Jacksonville State University, Jacksonville	29 Mar 2010
GA Epsilon	Wesleyan College, Macon	30 Mar 2010
FL Gamma	Southeastern University, Lakeland	31 Mar 2010
MA Beta	Stonehill College, Easton	8 Apr 2011
AR Beta	Henderson State University, Arkadelphia	10 Oct 2011

PA Tau	DeSales University, Center Valley	29 Apr 2012
TN Zeta	Lee University, Cleveland	5 Nov 2012
RI Beta	Bryant University, Smithfield	3 Apr 2013
SD Beta	Black Hills State University, Spearfish	20 Sept 2013
FL Delta	Embry-Riddle Aeronautical University, Daytona Beach	22 Apr 2014
IA Epsilon	Central College, Pella	30 Apr 2014
CA Eta	Fresno Pacific University, Fresno	24 Mar 2015
OH Theta	Capital University, Bexley	24 Apr 2015
GA Zeta	Georgia Gwinnett College, Lawrenceville	28 Apr 2015
MO Xi	William Woods University, Fulton	17 Feb 2016
IL Kappa	Aurora University, Aurora	3 May 2016
GA Eta	Atlanta Metropolitan University, Atlanta	1 Jan 2017
CT Gamma	Central Connecticut University, New Britain	24 Mar 2017
KS Eta	Sterling College, Sterling	30 Nov 2017
NY Sigma	College of Mount Saint Vincent, The Bronx	4 Apr 2018
PA Upsilon	Seton Hill University, Greensburg	5 May 2018
KY Gamma	Bellarmino University, Louisville	23 April 2019